NEW SYLLABUS MATHEMATICS 4 (6th Edition)
Specific Instructional Objectives (SIOs)

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SET A

This file contains a specified/suggested teaching schedule for the teachers.

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Week	Topic	Specific Instructional Objectives	Exercises	Maths Communication	Maths Investigation	Problem Solving	NE	IT	Resources
Term 1 **Week 1, 2, 3 & 4**	**Chapter 1** **Graphical Solution of Equations**	• Construct a table of values of x and y for (i) a cubic function, $y = ax^3 + bx^2 + cx + d$, (ii) a reciprocal function, $y = \frac{a}{x}$ and $y = \frac{a}{x^2}$, (iii) an exponential function, $y = a^x$, and plot the graphs of these functions on a piece of graph paper.	1a	Where do you find uses of graphs in everyday life situations?	Use Graphmatica to see the shape of graphs and to solve equations graphically.	Pg 26, 31 Pg 17, 37		Graph-matica: Pg 3, 6, 9, 10, 19, 22, 25, 28	Textbook
		• Find the value(s) of x for a given value of y and the value of y for a given value of x from the graphs above.	1a						
		• Sketch graphs of quadratic functions of the form $y = ax^2$, $y = \pm(x-a)(x-b)$ and $y = \pm(x-p)^2 + q$ where a, b, p and q are constants.	1b						
		• Draw the graphs of a quadratic function and use it to solve related quadratic equations graphically.	1b						
		• Draw the graphs of cubic, reciprocal and exponential functions and use them to solve related equations graphically.	1b						
Term 1 **Week 5 & 6**	**Chapter 2** **Further Graphs and Graphs Applied to Kinematics**	• Convert speeds from km/h to m/s and vice versa.	2a	Pg 43, 45 **Just For Fun** Ask for various answers and let pupils explain how they got them.		Pg 42, 58, 68, 70	Pg 50 Exercise 2a Q3 & Q5 Pg 68 Review Questions 2 Q4	Graph-matica: Pg 47-48	Textbook
		• Find the gradients of a curve by drawing a tangent to the curve.	2a						
		• Draw the distance-time graph from given information and use it to find the velocity and solve related problems.	2a						

Week	Topic	Specific Instructional Objectives	Exercises	Maths Communication	Maths Investigation	Problem Solving	NE	IT	Resources
		• Interpret a velocity-time graph and use it to find the distance moved by calculating the area under the curve; find the instantaneous acceleration at any point of time by finding the gradient of the tangent of the velocity-time graph at that time.	2b						
		• Draw a velocity-time graph from given information and use it to solve problems on distance, average speed and acceleration.	2b						
		• Solve problems relating to graphs in practical situations.	2b						
Term 1 **Week 7, 8, 9 & 10**	**Chapter 3** **Vectors in Two Dimensions**	• Differentiate between scalars and vectors and give two examples of each.	3a	Pg 75	Pg 103	Pg 79			Textbook
		• Represent a vector using proper terminologies and notations.	3a						
		• Define and identify equal vectors.	3a						
		• Define and identify negative of a vector and the zero vector.	3a						
		• Express a vector in column vector form.	3b						
		• Find the magnitude and direction of a vector in column vector form.	3c						
		• Use triangle law of vector addition to find the sum of and difference between two vectors.	3d						
		• Multiply a column vector by a scalar.	3e						

Week	Topic	Specific Instructional Objectives	Exercises	Maths Communication	Maths Investigation	Problem Solving	NE	IT	Resources
		• Express a given vector in terms of two component vectors. • Define position vector. • Find the resultant of two position vectors.	3e						
Term 2 **Week 1 & 2**	**Chapter 4** **Standard Deviation and Mean**	• Find the mean of a given grouped data. • Calculate the standard deviation of a set of data. • Calculate the standard deviation of a set of grouped data. • Able to comment and compare the performance of two sets of data based on the mean and standard deviation.	4a 4b 4b 4b	Discuss how some statistics may be manipulated or misrepresented. What are the properties of standard deviation and how they are used in everyday situations.				Excel: Pg 144	Textbook
Term 2 **Week 3, 4 & 5**	**Chapter 5** **Cumulative Frequency Distribution**	• Construct a cumulative frequency table from a given frequency distribution table. • Draw a cumulative frequency curve and use it to estimate the number or percentage of particular participants exceeding or falling short of a figure. • Find the median, lower and upper quartiles and percentiles from a cumulative frequency curve and use them to find inter-quartile range and solve other related problems. • Able to comment and compare the performance of two sets of data based on the median and inter-quartile range of the data.	5a 5a 5b 5b		Pg 174	Pg 189, 206		Excel: Pg 182-183	Textbook

Week	Topic	Specific Instructional Objectives	Exercises	Maths Communication	Maths Investigation	Problem Solving	NE	IT	Resources
		• Draw a box-and-whisker plot from a set of data. • Able to comment and compare the performance of two sets of data based on box-and-whisker plots of the sets of data.	5c 5c						
Term 2 **Week 6, 7 & 8**	**Chapter 6** **More on Probability**	• Define the classical definition of probability of an event E occurring as $P(E)=\frac{\text{No. of outcomes favouable to the occurence of E}}{\text{Total number of equally likely outcomes}}$ • List the elements in the sample space of an experiment. • Use the possibility diagrams to list the sample space of simple combined events. • Use the tree diagrams to list the sample space of simple combined events. • Perform calculation using the addition law to find the probability of mutually exclusive events. • Perform calculation using the multiplication law to find the probability of independent events. • State that for any event E, $0 \le P(E) \le 1$. • P(E)=0 if and only if the event E cannot possibly occur. • P(E)=1 if and only if the event E will certainly occur. • State the rule P(E) = 1 – P(E') where E and E' are	6a 6b 6c 6d 6d 6e	Discuss "Is it worthwhile to gamble? What are the odds? Is it better to bet on 4-digit 'BIG' or 'SMALL'?" Refer to Pg 362 and TG.	Pg 224, 231, 232,	Pg 221, 234-235,			Textbook

Week	Topic	Specific Instructional Objectives	Exercises	Maths Communication	Maths Investigation	Problem Solving	NE	IT	Resources
		complementary events. • Use all the above theory to solve problems involving two or more events.							
Term 3 **Week 1 to 7**	**Chapter 7** **Revision**		7a onwards	Pg 349 : Should we be proud of ourselves for being great gamblers? Pg 353 : Can you give concrete examples where statistics are being distorted?			Pg 306, 317 322, 326, 327		Textbook

Chapter 1

Secondary 4 Mathematics
Chapter 1 Graphical Solutions of Equations

ANSWERS FOR ENRICHMENT ACTIVITIES

Just For Fun (pg 27)

From line 4 to line 5; the division of $(a - b)$ on both sides is unacceptable as $(a - b) = 0$.

Just For Fun (pg 37)

(a) $2^2 - 1 = 3$ (b) $2^3 - 1 = 7$ (c) $2^4 - 1 = 15$ (d) $2^{10} - 1 = 1\,023$

(e) $2^n - 1$

Secondary 4 Mathematics
Chapter 1 Graphical Solutions of Equations

GENERAL NOTES

Before plotting graphs of functions, revise with the pupils the general method on the choice of scales for the straight line graph and the quadratic graphs that they had learned in Secondary 2 and the plotting of travel graphs and conversion graphs that they learned in Secondary 3. Remind them to label the graphs clearly. Pupils should be encouraged to draw the curves free hand as well as to use curved rules to assist them.

There are many opportunities for teachers and pupils to explore this chapter using softwares such as Graphmatica, Winplot and others. These tools will help the teachers to plot the graphs quickly and to illustrate the solution of solving equations and leaving more time for the technique of manipulating the equation to find the correct graph to add onto an existing graph to solve a particular equation.

It will be good to ask pupils to remember the general shapes of quadratic, cubic, reciprocal and exponential graphs. This will enable pupils to identify and rectify errors when they plot a few points wrongly and the shapes of their graphs look odd. They are encouraged to plot a few of these graphs using Graphmatica or Winplot.

The concept that the points of intersection of two graphs give the solution of a pair of simultaneous equations is important and this can be illustrated by solving a pair of linear simultaneous equations and then plotting the graphs of these two linear equations to verify the results.

For manipulation of solutions of equations by graphical means, many examples should be used to illustrate the point. Example 10 in the text will be a good guide. Besides the equations discussed in Example 10, equations such as

$4x^2 + 7x - 3 = 0$
$2x^2 - 2x - 7 = 0$

and others could also be used.

The sketching of quadratic graphs is new in this syllabus. The Explorations of the Graphs of the form $y = \pm(x-a)(x-b)$ and $y = \pm(x-p)^2 + q$ where p and q are constants provided in the textbook are worthwhile activities for the pupils to explore.

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Name: _____ ( ) Date: _____

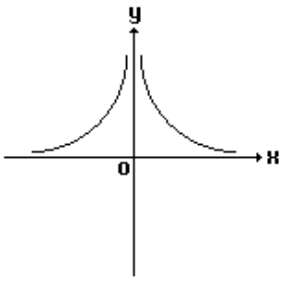
Time allowed: min

Class: _____ Marks: /10

Secondary 4 Multiple-Choice Questions
Chapter 1 Graphical Solutions to Equations

1. Which of the following could be the graph of $y = \frac{a}{x}$, where $a < 0$.

(A)



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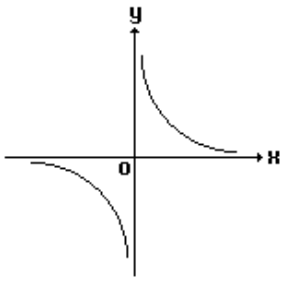

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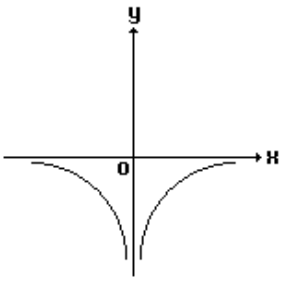

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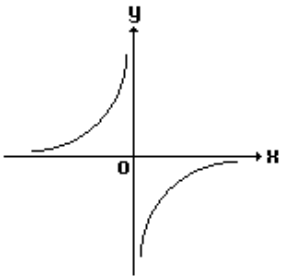

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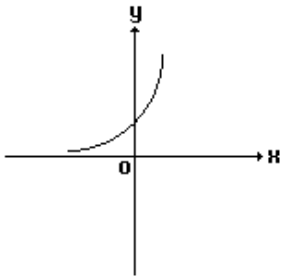

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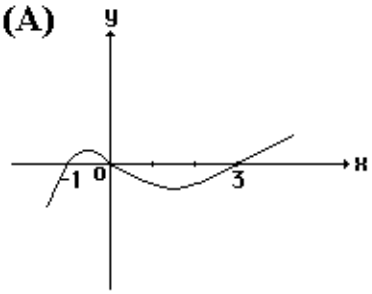
2. Which of the following could be the graph of $y = x^2(x - 3)$?

(A)



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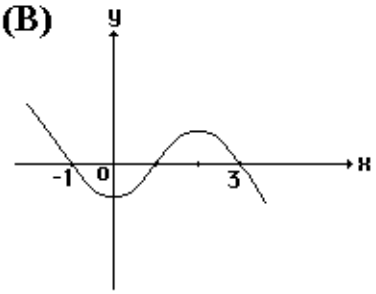

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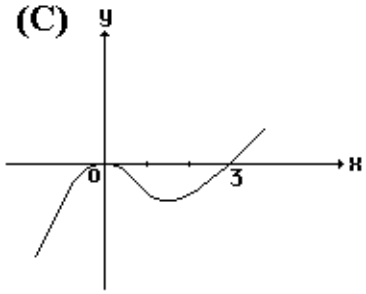

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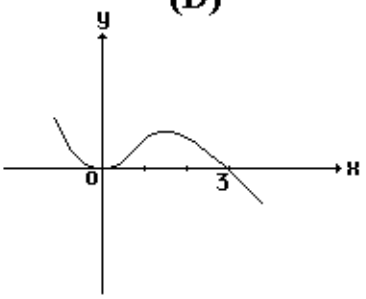

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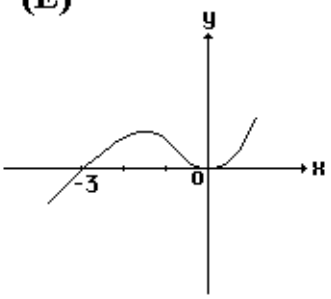

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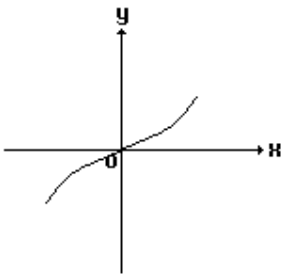
3. Which of the following could be the graph of $y = x^3 - 1$?

(A)



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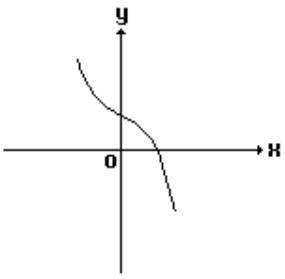

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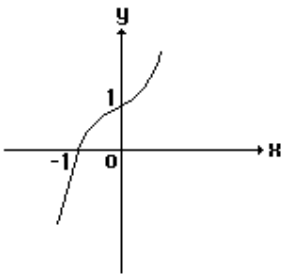

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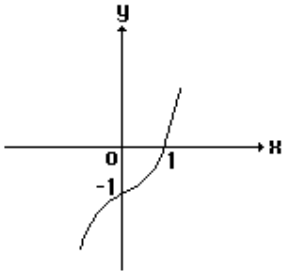

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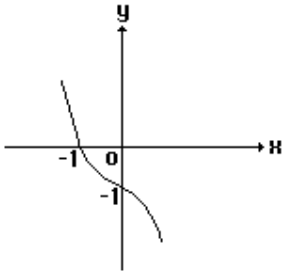

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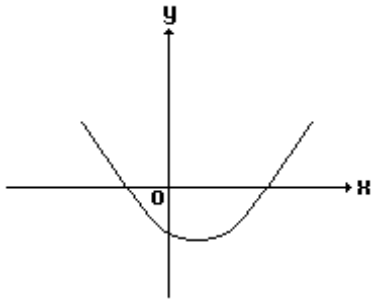
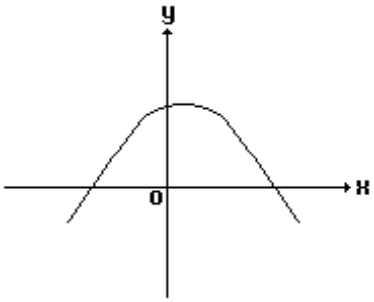
4. The diagram shows the graph of $y = ax^2 + bx + c$.
Determine whether a and c are positive or negative.



(A) $a > 0$ and $c > 0$
(B) $a > 0$ and $c < 0$
(C) $a < 0$ and $c < 0$
(D) $a < 0$ and $c > 0$
(E) cannot be determined.

( )

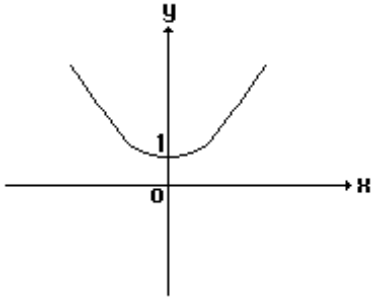
5. The diagram shows the graph of $y = ax^2 + bx + c$.
Determine whether a and c are positive or negative.



(A) $a > 0$ and $c > 0$
(B) $a > 0$ and $c < 0$
(C) $a < 0$ and $c < 0$
(D) $a < 0$ and $c > 0$
(E) cannot be determinec

( )

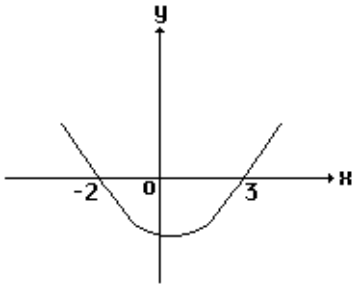
6. The diagram shows the graph $y = 2x^2 + 1$.
To solve the equation $2x^2 + x - 5 = 0$ graphically, which of the following graphs must we draw?



(A) $y = x - 5$
(B) $y = x - 6$
(C) $y = 6 - x$
(D) $y = \frac{1}{2}x - 3$
(E) none of the above.

( )

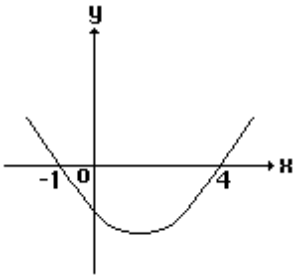
7. The diagram shows the graph $y = x^2 - x - 6$.
To solve the equation $x^2 - 5x - 3 = 0$ graphically, which of the following graphs must be drawn?



(A) $y = 4x - 3$ (D) $y = -4x - 3$
(B) $y = 4x + 3$ (E) $y = 5x - 3$
(C) $y = -4x + 3$

( )

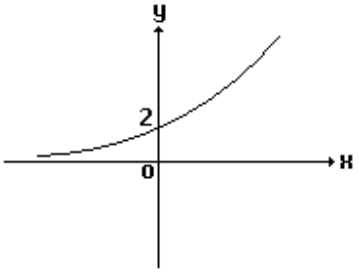
8. The diagram shows the graph of $y = x^2 - 3x - 4$.
To solve the equation $2x^2 - 4x = 3$ graphically, which of the following graphs must be drawn?



(A) $y = x + 2\frac{1}{2}$ (D) $y = -x - 2\frac{1}{2}$
(B) $y = -2x + 5$ (E) none of the above.
(C) $y = -2x - 5$

( )

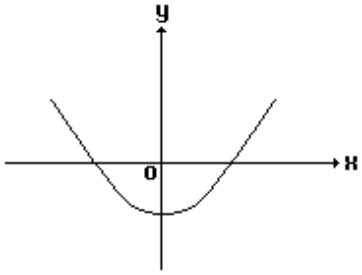
9. The diagram shows the graph of $y = 2^x + 1$.
To solve the equation $2^x = 3 - x$ graphically, which of the following graphs must be drawn?



(A) $y = 3 - x$ (D) $y = 2 - x$
(B) $y = x - 4$ (E) $y = 4 - x$
(C) $y = x + 1$

( )

10. The diagram shows the graph of $y = x^2 - 4$. Which of the following graphs must you draw in order to solve the equation $x^3 - 4x - 3 = 0$?



(A) $y = \frac{3}{x}$ (D) $y = 1 - \frac{3}{x}$
(B) $y = -\frac{3}{x}$ (E) none of the above.
(C) $y = \frac{2}{x}$

( )

Answers

1. D
2. C
3. D
4. B
5. D
6. C
7. A
8. D
9. E
10. A

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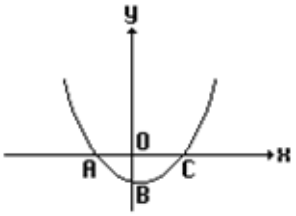
Name: _____ ( ) Date: _____

Time allowed: min

Class: _____ Marks:

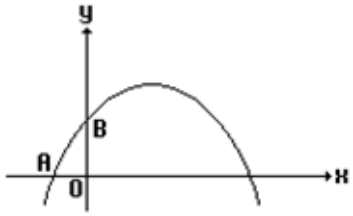
Secondary 4 Mathematics Test
Chapter 1 Graphical Solutions to Equations

1. The figure shows the curve $y = (2x + 1)(4x – 3)$.
Write down the coordinates of A, B and C. [3]



2. The diagram is a sketch of the curve
$y = (1 + 2x)(3 – x)$.
Write down

(a) the coordinates of A and B, [2]
(b) the equation of the line of symmetry of the curve. [1]



3. The following is a table of values for the function $y = x^2 + \frac{10}{x} - 6$.

x	0.5	1	1.5	2	3	4
y	10.2	5	h	3	6.3	12.5

(a) Find the value of h. [1]

(b) Choose suitable scales to draw the graph of $y = x^2 + \frac{10}{x} - 6$ for $0.5 \le x \le 4$. [4]

(c) Use your graph to find the value of y when $x = 2.4$. [1]

4. The following is a table of values for the function $y = -x^3 + 3x^2 + 1$.

x	-1	-0.5	0	0.5	1	1.5	2	2.5	3
y	5	1.88	1	h	3	4.37	5	4.12	k

(a) Calculate the values of h and k. [1]

(b) Using a scale of 4 cm to represent 1 unit on both axes, draw the graph of $y = -x^3 + 3x^2 + 1$ for $-1 \leq x \leq 3$. [4]

(c) Use your graph to find the values of y when
(i) $x = 0.2$ **(ii)** $x = 1.8$ [2]

(d) Use your graph to find the values of x when $y = 2.3$. [2]

5. Answer the whole of this question on a piece of graph paper. The variables x and y are connected by the equation $y = x + \frac{9}{x} - 5$. The table below shows some corresponding values of x and y.

x	1	2	3	4	5	6	7
y	5.0	p	1.0	1.3	q	2.5	3.3

(a) Calculate the values of p and q. [1]

(b) Using a scale of 2 cm to represent 1 unit on both axes, draw the graph of $y = x + \frac{9}{x} - 5$ for $1 \leq x \leq 7$. [4]

(c) Use your graph to find
(i) the value of y when $x = 4.4$, [1]
(ii) the values of x when $y = 2.8$, [2]
(iii) the value of x when y is a minimum. [1]

6. Copy and complete the given table of values for the function $y = 6x - x^2 + 2$.

x	– 1	0	1	2	3	4	5	6	7
y		2	7						

[1]

(a) Using a scale of 2 cm to represent 1 unit on the x-axis and 1 cm to represent 1 unit on the y-axis, draw the graph of $y = 6x - x^2 + 2$ for $1 \leq x \leq 7$. [4]

(b) Use your graph to find the values of

(i) y when $x = 1.4$, [1]

(ii) x when $y = 4.5$. [2]

7. The variables x and y are connected by the equation $y = 2x^2 - 3x - 4$. The table below shows some corresponding values of x and y.

x	– 3	– 2	– 1	0	1	2	3	4	5
y	23	10	1	– 4	– 5	– 2	5	p	q

(a) Calculate the values of p and q. [1]

(b) Taking 2 cm to represent 1 unit on the x-axis and 2 cm to represent 5 units on the y-axis, draw the graph of $y = 2x^2 - 3x - 4$ for $-3 \leq x \leq 5$. [4]

(c) Use your graph to find the values of

(i) y when $x = -1.8$, [1]

(ii) x when $y = 8$. [2]

8. The values of x and y are connected by the equation $y = 17x^2 - 120x + 200$. Copy and complete the table of values for $y = 17x^2 - 120x + 200$. [1]

x	1	2	3	4	5	6
y	97	28	– 7		25	

(a) Using a scale of 2 cm to 1 unit on the x-axis and 2 cm to 40 units on the y-axis, draw the graph of $y = 17x^2 - 120x + 200$ for $1 \leq x \leq 6$. [4]

(b) Find from your graph the smallest value of y and state the corresponding value for x. [2]

(c) Use your graph to find

(i) the value of y when $x = 2.5$, [1]

(ii) the value of x when $y = 20$. [2]

9. The following is a table of values for the function $y = x\,(2x^2 - 7x + 6)$.

x	0	0.25	0.5	0.75	1	1.25	1.5
y	0	1.09	h	1.41	1	0.47	k

(a) Calculate the values of h and k. [2]

(b) Using a scale of 8 cm to represent 1 unit on both x and y - axes, draw the graph of $y = x\,(2x^2 - 7x + 6)$ for $0 \leq x \leq 1.5$. [3]

(c) Use your graph to find the values of y when

(i) $x = 0.6$ **(ii)** $x = 1.4$. [2]

(d) Use your graph to find the values of x when $x = 1.25$. [2]

10. Answer the whole of this question on a sheet of graph paper. The variables x and y are connected by the equation $y = 20 + 4x - 3x^2$. The table below shows some corresponding values of x and y.

x	– 3	– 2	– 1	0	1	2	3	4	5
y	h	0	13	20	21	16	5	k	– 35

(a) Calculate the values of h and k. [1]

(b) Using a scale of 2 cm to represent 1 unit on the x-axis and 2 cm to represent 10 units on the y-axis, draw the graph of $y = 20 + 4\,x - 3x^2$ for $-3 \leq x \leq 5$. [4]

(c) Use your graph to find the value of y when

(i) $x = -1.4$ **(ii)** $x = 3.3$. [2]

(d) Use your graph to find the value of x when

(i) $y = 0$ **(ii)** $y = 15$. [3]

11. Answer the whole of this question on a sheet of graph paper. The variables x and y are connected by the equation $y = \frac{1}{2}x^2 + \frac{18}{x} + 3$. Some corresponding values of x and y are given in the table below.

x	1	1.5	2	2.5	3	3.5	4	4.5	5
y	21.5	16.1	11	h	13.5	11.3	15.5	k	19.1

(a) Calculate the values of h and k. [1]

(b) Using a scale of 2 cm to 1 unit on both axes, draw the graph of $y = \frac{1}{2}x^2 + \frac{18}{x} + 3$ for $1 \leq x \leq 5$. [4]

(c) Use your graph to find the values of
(i) y when $x = 2.2$ **(ii)** x when $y = 15$. [2]

12. The table below shows corresponding values of x and y for the function $y = 20 - 3x - \frac{60}{x}$.

x	2	3	4	5	6	7	8	9
y	– 16	– 9	– 7	p	q	– 9.6	r	– 13.7

(a) Calculate the values of p, q and r. [2]

(b) Using a scale of 2 cm to represent 1 unit on both axes, plot the graph of $y = 20 - 3x - \frac{60}{x}$ for $2 \leq x \leq 9$. [3]

(c) Use your graph to find the value of y when
(i) $x = 2.5$ **(ii)** $x = 5.4$. [2]

(d) Find the greatest possible value of y in this range and state the value of x when this occurs. [2]

13. The variables x and y are related by the equation $y = \frac{x}{4} + \frac{4}{x} + 4$. The table below shows some corresponding values of x and y.

x	1	2	3	4	5	6	7
y	8.25	h	6.08	6	k	6.17	6.32

(a) Find the value of h and k. [1]

(b) Using a scale of 2 cm to represent 1 unit on the x-axis and 4 cm to represent 1 unit on the y-axis, draw the graph of $y = \frac{x}{4} + \frac{4}{x} + 4$ for $1 \le x \le 7$. [4]

(c) Use your graph to find the values of y when

(i) $x = 1.5$ **(ii)** $x = 5.3$. [2]

(d) Use your graph to find the values of x when

(i) $y = 6.2$ **(ii)** $y = 7.5$. [3]

14. The diagram shows part of the graph of $y = 12 + x - x^2$. The graph cuts the x–axis at P and R, and the y–axis at Q.

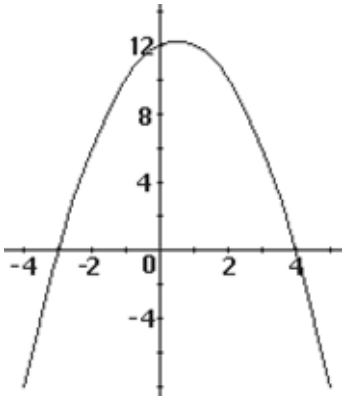
(a) Find the coordinates of P, Q and R. [3]

(b) Write down the equation of the line of symmetry of the graph $y = 12 + x - x^2$. [1]

(c) Find the maximum value of y. [1]

(d) Find the equation of the straight line that must be drawn on the diagram to solve the following equations graphically.

(i) $x^2 - x - 15 = 0$ (ii) $x^2 - 5 = 0$ [2]



15. Using the same scales and axes as in question 1, draw the graphs of $y = 3x + 4$ and $x + y = 10$. Find the co-ordinates of the point of intersection of the two graphs. [4]

16. Answer the whole of this question on a sheet of graph paper. The length of a solid cuboid is three times its width. Given that the width of the cuboid is x cm, and its volume 36 cm³, write down an expression for the height in terms of x. [1]

If the total surface area of the cuboid is y cm³, show that $y = 6x^2 + \frac{96}{x}$ [2]

The table below shows some corresponding values of x and y.

x	1	1.5	2	2.5	3	3.5	4
y	102	77.5	h	75.9	86	k	120

(a) Find the values of h and k. [2]

(b) Using a scale of 4 cm to represent 1 cm on the x–axis and 2 cm to represent 10cm² on the y–axis, draw the graph of $y = 6x^2 + \frac{96}{x}$ for $1 \le x \le 4$. [3]

(c) Use your graph to find

(i) the two possible values of x for which $y = 90$. [2]

(ii) the surface area of the cuboid when the length is 9.6 cm, [1]

(iii) the height of the cuboid when y is a minimum. [1]

17. Answer the whole of this question on a sheet of graph paper. The variable x and y are connected by the equation $y = 8 + 7x - x^2$. Some of the corresponding values of x and y are given in the following table .

x	–1	0	1	2	3	4	5	6	7	8
y	0	8	14	18	h	20	18	14	8	0

(a) Calculate the value of h. [1]

(b) Using a scale of 2 cm to represent 1 unit on the x–axis and 1 cm to represent 1 unit on the y–axis, draw the graph of $y = 8 + 7x - x^2$ for $-1 \le x \le 8$. [4]

(c) Write down the equation of the line of symmetry of the curve $y = 8 + 7x - x^2$. [1]

(d) By drawing a suitable straight line on your graph, find the range of values of x for which $5x \ge x^2 + 4$. [3]

(e) By drawing another straight line on your graph, solve the equation $7x - x^2 = 2x - 3$. [3]

18. The following table gives some corresponding values of x and y connected by the equation $2 + x + \frac{32}{x^2}$.

x	2	2.5	3	4	5	6	7
y	12	h	8.56	8	8.28	8.89	9.65

(a) Calculate the value of h. [1]

(b) Using a scale of 2 cm to represent 1 unit on both axes, draw the graph of $y = 2 + x + \frac{32}{x^2}$ for $2 \le x \le 7$. [3]

(c) Find the least value of y in the given range and state the corresponding value of x. [2]

(d) By drawing a tangent, find the gradient of the curve at the point $x = 5.4$ [2]

(e) By drawing a suitable straight line graph on the same graph paper, solve the equation $x^3 + 32 = 7x^2$. [2]

(f) Find the range of values of x for which $\frac{32}{x^2} < 5 - \frac{2}{3}x$. [2]

19. The variables x and y are connected by the equation $y = 2x\sqrt{12 - x^2}$. The table below shows some corresponding values of x and y for $y = 2x\sqrt{12 - x^2}$.

x	1	1.25	1.5	1.75	2	2.25	2.5	2.75	3
y	6.6	8.1	9.4	10.5	11.3	p	12.0	q	10.4

(a) Calculate the value of p and q. [2]

(b) Using a scale of 4 cm to represent 1 unit on the x-axis and 2 cm to represent 1 unit on the y–axis, draw the graph of $y = 2x\sqrt{12 - x^2}$ for $1 \le x \le 3$. [4]

(c) Find the gradient of the curve at the point where $x = 1.5$ by drawing a tangent. [2]

(d) By drawing a suitable straight line, find the values of x for which $x\sqrt{12 - x^2} = x + 3$. [2]

(e) Find the greatest possible value of y and the corresponding value of x in the range. [2]

20. Answer the whole of this question on a sheet of graph paper.
The table below gives the x and y coordinates of some points which lie on a curve.

x	–2	–1	0	1	2	3	4	5	6
y	12	5	0	–3	–4	–3	0	5	12

(a) Using a scale of 2 cm to represent 1 unit on the x–axis and 1 cm to represent 1 unit on the y–axis, plot the coordinates of the given points and draw a smooth curve through them. [3]

(b) Write down the equation of the line of symmetry of the curve. [1]

(c) The points (l.4, k) and (h, k) lie on the curve. Use your graph to find the values of h and k. [2]

(d) The values of x and y are related by the equation $y = px^2 + qx$. Use the fact that (2, –4) lie on the curve to show that $4p + 2q = -4$. [1]

(e) Use the fact that (–l, 5) also lie on the curve to derive another equation connecting p and q. Hence calculate the values of p and q using the relation derived in (d). [3]

(f) By drawing a tangent, find the gradient of the curve at the point $x = 0.5$. [2]

21. Answer the whole of this question on a sheet of graph paper.
The variables x and y are connected by the equation $y = x^2 + x - 6$, and some corresponding values are given in the following table.

x	–4	–3	–2	–1	0	1	2	3
y	6	h	–4	–6	–6	k	0	6

(a) Calculate the values of h and k. [2]

(b) Taking 2 cm to represent 1 unit on the x–axis and 1 cm to represent 1 unit on the y–axis, draw the graph of $y = x^2 + x - 6$ for $-4 \le x \le 3$. [3]

(c) By drawing a suitable straight line on your graph, find the values of x satisfying the equation $x^2 + x - 6 = -(x + 4)$. [3]

(d) From your graph, estimate the range of values of x for which $x^2 + x - 6 > 1.5$. [2]

(e) By drawing a tangent, find the gradient of the graph at the point where $x = 1$. [2]

22. Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation $y = \frac{1}{2}x - \frac{2}{x}$. Some corresponding values of x and y are given in the following table.

x	0.5	1	1.5	2	2.5	3	3.5	4
y	−3.75	−1.5	−0.58			0.83		1.5

Copy and complete the table above. [2]

Taking 2 cm to represent 1/2 unit on the x–axis and 2 cm to represent 1 unit on the y–axis, draw the graph of $y = \frac{1}{2}x - \frac{2}{x}$ for the range $0.5 \le x \le 4$. [3]

(a) From your graph, find

(i) the value of y when $x = 1.8$,

(ii) the value of x when $y = 0.5$. [2]

(b) Find, by drawing a tangent, the gradient of the graph at the point $x = 1$. [2]

(c) Obtain one solution of the equation $x^2 + 2x = 4$ from your graph. [3]

23. Answer the whole of this question on a sheet of graph paper.

Copy and complete the table of values of $y = 5x + \frac{60}{x} - 28$. [1]

x	1.5	2	2.5	3	4	5	6	7	8
y	19.5	12		7		9	12	15.6	19.5

Using a scale of 2 cm each to represent 1 unit of x and 2 units of y, draw the graph of $y = 5x + \frac{60}{x} - 28$ for $1.5 \le x \le 8$. [3]

(a) Use your graph to find

(i) the least value of y, [1]

(ii) the range of values of x for which y is less than 9. [2]

(b) By drawing a tangent, find the gradient of the curve when $x = 6$. [2]

(c) By drawing a suitable straight line on the same axes, use your graph to find the solutions of the equation $6x + \frac{60}{x} - 48 = 0$. [3]

24. Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation $y = 35 - 3x - \frac{60}{x}$. Some of the corresponding values of x and y are given in the following table.

x	2	2.5	3	4	5	6	7	8	9
y	–1	h	6	8	8	7	5.4	k	1.3

(a) Calculate the values of h and k. [1]

(b) Using a scale of 2 cm to represent 1 unit on both the x and y axes, draw the graph of $y = 35 - 3x - \frac{60}{x}$ for $2 \le x \le 9$. [3]

(c) Find the largest value of y and the value of x that corresponds to this greatest value of y. [2]

(d) By drawing a tangent, find the gradient of the curve at the point $x = 6.5$. [2]

(e) By drawing a suitable straight line on the graph, solve the equation $3x + \frac{60}{x} = 30$. [2]

(f) By drawing a suitable straight line on the same graph, find the range of values of x for which $7x^2 - 66x + 120 \le 0$. [2]

25. The following is a table of values for the graph of $y = \frac{1}{5}x\,(x + 2)\,(x - 2)$.

x	-3	-2	-1	0	0.5	1	2	2.5	3
y	-3	*0*	0.6	0	-0.38	p	q	*1.13*	3

(a) Calculate the values of p and q. [2]

(b) Using scale of 2 cm to represent 1 unit on the x–axis and 1 cm to represent 1 unit on the y–axis, draw the graph of $y = \frac{1}{5}x\,(x + 2)\,(x - 2)$ for $-3 \le x \le 3$. [3]

(c) By drawing a tangent, find the gradient of the curve $y = \frac{1}{5}x\,(x + 2)\,(x - 2)$ at the point $x = 2$. [2]

(d) Use your graph to find the values of x for which $y = \frac{1}{2}x$ meets the curve. [2]

(e) By drawing a suitable straight line on the same graph, solve the equation $25x + 25 = x^3 - 4x$. [3]

26. The following is a table of values for the graph of $y = x^2 - 2x - 1$.

x	–2	–1	0	1	2	3	4
y	7	2	–1	–2	–1	2	7

(a) Using a scale of 2 cm to represent 1 unit on the x–axis and 1 cm to represent 1 unit on the y–axis, draw the graph of $y = x^2 - 2x - 1$ for $-2 \le x \le 4$. [3]

(b) By drawing a tangent, find the gradient of the curve $y = x^2 - 2x - 1$ at the point $x = 2.5$. [2]

(c) By drawing a suitable straight line using the same scales and axes, find the solutions of each of the equations below from your graph.

(i) $x^2 - 2x = 1$ [1]

(ii) $x^2 - 3x = 2$ [2]

(iii) $x^2 - x = 5$ [2]

(d) By drawing another straight line on the graph paper, find the range of values of x for which $x^2 - 3x \le 1$. [2]

27. The variables x and y are related by the equation $y = \dfrac{6x}{x+2}$. The table below shows some corresponding values of x and y for $y = \dfrac{6x}{x+2}$.

x	–0.5	0	1	2	3	4	5	6
y	h	0	2	3	3.6	4	4.3	k

(a) Calculate the values of h and k. [1]

(b) Using a scale of 2 cm to represent 1 unit on both x and y axes, draw the graph of $y = \dfrac{6x}{x+2}$ for $-0.5 \le x \le 6$. [3]

(c) By drawing a suitable tangent, find the gradient of the curve at the point $x = 1.5$. [2]

(d) From your graph, find the values of x in the range $-0.5 \le x \le 6$ for which

(i) $\dfrac{6x}{x+2} < 2.5$ [2]

(ii) $\dfrac{6x}{x+2} > x+1$ [2]

(e) Find the solution of the equation (in the range $-0.5 \le x \le 6$), $6x = (x + 2)(4 - x)$, from your graph, by drawing a suitable straight line. [2]

28. Answer the whole of this question on a sheet of graph paper.

The values of x and y are connected by the equation $y = \frac{3(x-1)(x+2)}{(x-4)}$. Some corresponding values of x and y are given in the following table.

x	–5	–4	–3	–2	–1	0	1	2
y	–6		–1.71			1.5		–6

(a) Find the missing values of y. [2]

(b) Taking 2 cm to represent 1 unit on each axis, draw the graph of $y = \frac{3(x-1)(x+2)}{(x-4)}$ for the range of $-5 \le x \le 2$. [4]

(c) Use the graph to solve the equation $3(x-1)(x+2) = x-4$. [2]

(d) By adding a suitable straight line to your graph, estimate the range of values of x for which $\frac{6(x-1)(x+2)}{(x-4)} \le 1-x$ [2]

(e) By drawing a suitable tangent, estimate the gradient of the curve at the point $x = -3$. [2]

29. (a) Copy and complete the following table of values for $y = 5 + \frac{16}{2^x}$

(Given values of y correct to 1 decimal place.) [1]

x	0	$\frac{1}{2}$	1	2	3	4	5	6
y	21		13	9	7	6		5.3

(b) Using a scale of 2 cm to represent 1 unit on the x–axis and 1 cm to represent 1 unit on the y–axis, draw the graph of $y = 5 + \frac{16}{2^x}$ for $0 \le x \le 6$. [3]

(c) Use your graph to solve the equations

(i) $5 + \frac{16}{2^x} = 10$,

(ii) $\frac{16}{2^x} = 12$ [3]

(d) On the same axes, draw the graph of $y = 4x + 6$. Write down the coordinates of the point where the graph of $y = 4x + 6$ meets the graph of $y = 5 + \frac{16}{2^x}$ [3]

(e) By drawing a tangent at the point where $x = 3$, find the gradient of the curve $y = 5 + \frac{16}{2^x}$ at the point (3, 7). [2]

30. Given that $y = 3x - 2x^2 + 1$, copy and complete the following table. [2]

x	–2	$-1\frac{1}{2}$	–1	0	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3
y		–8		1			–1		

Using 2 cm to represent 1 unit on the x-axis and 1 cm to represent 1 unit on the y-axis, draw the graph of $y = 3x - 2x^2 + 1$ for $-2 \le x \le 3$. [4]

Use your graph to find

(a) the values of y when $x = 0.5, 1.4, 2.8$, [2]

(b) the values of x when $y = 0, -4$. [1]

31. Copy and complete the following table for $y = x^3 + 2x^2 - 5x + 5$. [2]

x	– 4	–3	–2	–1	0	1	2	3
y	–7	11		11	5	3	11	

(a) Using a scale of 2 cm for 1 unit on the x-axis and 2 cm for 5 units on the y-axis, draw the graph of $y = x^3 + 2x^2 - 5x + 5$ for $-4 \le x \le 3$. [4]

(b) Use your graph to find the values of

(i) y when $x = -2.6$ and 1.7, [1]

(ii) x when $y = 7$ and 1.5. [1]

32. The following table gives values of x and y for $y = 16(0.5)^x$.

x	0	1	2	3	4	5	6	7	8
y	16	8	4	2	1	0.5	0.25	p	q

(a) Calculate the value of p and q. [2]

(b) Using a scale of 2 cm to represent 1 unit on the x-axis and 1 cm to represent 1 unit on the y-axis, plot the graph of $y = 16(0.5)^x$ for $0 \le x \le 8$. [3]

(c) Use your graph to find the values of

(i) x when $y = 3.5$ and 0.6, [1]

(ii) y when $x = 1.4$ and 4.7. [1]

33. Copy and complete the following table for $y = 4x + \frac{60}{x} - 30$. [2]

x	1.5	2	2.5	3	4	5	6	7	8
y	16		4		1		4		9.5

Using a scale of 2 cm to represent 1 unit on the x-axis and 1 cm to represent 1 unit on the y-axis, plot the graph of $y = 4x + \frac{60}{x} - 30$ for $1.5 \le x \le 8$. [3]

Use your graph to answer the following:

(a) What is the value of y when $x = 3.5$? [1]

(b) What are the possible values of x when $y = 3$? [2]

(c) Find the gradient of the curve at the point $x = 3$ by drawing a suitable tangent. [2]

34. Copy and complete the following table for $y = \frac{2x^2 - 3}{x + 5}$ for $-2 \le x \le 2.5$, giving values correct to 2 decimal places. [2]

x	–2	–1.5	–1	–0.5	0	0.5	1	1.5	2	2.5
y	1.67	0.43		–0.56		–0.46	–0.17		0.71	1.27

Using a scale of 4 cm to represent 1 unit on the x-axis and 4 cm to represent 0.5 unit on the y-axis, plot the graph of $y = \frac{2x^2 - 3}{x + 5}$ for $-2 \le x \le 2.5$. [3]

Use your graph to find

(a) the roots of the equation $\frac{2x^2 - 3}{x + 5} = 0$, [1]

(b) the range of values of x for which $\frac{2x^2 - 3}{x + 5} < 0.5$, [2]

(c) the gradient of the curve at the point $x = -1$ by drawing a suitable straight line. [1]

35. Given that $y = x(8 - x)(12 - x)$, copy and complete the following table for $y = x(8 - x)(12 - x)$. [2]

x	0	1	2	3	4	5	6	7	8
y	0	77		135				35	0

Using a scale of 2 cm to represent 1 unit on the x-axis and 2 cm to represent 20 units on the y-axis, draw the graph of $y = x(8 - x)(12 - x)$ for $0 \le x \le 8$. [3]

Use your graph to find

(a) the solution of the equation $x(8 - x)(12 - x) = 95$, [1]

(b) the values of x in this range for which $y \ge 110$, [2]

(c) the gradient of the curve at the point $x = 4.5$. [1]

36. The following table gives the corresponding values of x and y connected by the equation $y = 1 + \frac{2}{x} + x^2$.

x	0.1	0.3	0.5	1	1.5	2	2.5	3	3.5
y	21.01	7.76	5.25	4	p	6	q	10.67	r

(a) Calculate the values of p, q and r, giving your answer correct to 2 decimal places. [3]

(b) Using a scale of 4 cm for 1 unit on the x-axis and 1 cm for 1 unit on the y-axis, plot the graph of $y = 1 + \frac{2}{x} + x^2$ for $0.1 \leq x \leq 3.5$. [4]

(c) By drawing a tangent, find the gradient of the curve at the point $x = 2$. [1]

(d) By drawing suitable straight line graphs on the same graph paper, solve the equations

(i) $\frac{2}{x} + x^2 = 8$, **(ii)** $\frac{2}{x} + x^2 - 2x = 3$. [2]

(e) Find the range of values of x for which $\frac{2}{x} + x^2 > 6$ by drawing another straight line. [1]

37. The following table gives corresponding values of x and y which are connected by the equation $y = \frac{1}{12}(1.54)^x$.

x	0	1	2	3	4	5	6	7	8
y	0.08	0.13	0.20	0.30	h	0.72	k	1.71	2.64

(a) Calculate the value of h and k correct to 2 decimal places. [2]

(b) Using a scale of 2 cm for 1 unit on the x-axis and 8 cm for 1 unit on the y-axis, draw the graph of $y = \frac{1}{12}(1.54)^x$ for $0 \leq x \leq 8$. [3]

(c) Use your graph to solve the equations

(i) $(1.54)^x = 6$, (ii) $(1.54)^x = 2x + 3$. [2]

(d) Find the gradient of the curve at the point $x = 4$ by drawing a straight line. [1]

Answers

1. (a) $1\frac{1}{4}$ (b) $\frac{3x}{x-2}$, $x \neq 2$

2. (a) $A\,(-\frac{1}{2}\,, 0)$, $B\,(0, 3)$ (b) $x = 1\frac{1}{4}$

3. (a) 2.9 (c) 3.9

4. (a) $h = 1.63,\ k = 1$ (c) (i) 1.1 (ii) 4.9 (d) $x = -0.6,\ 0.75,\ 2.85$

5. (a) $p = 1.5,\ q = 1.8$ (c) (i) 1.4 (ii) 1.4, 6.4 (iii) 3

6. – 5, 10, 11, 10, 7, 2, – 5 (b) (i) 8.4 (ii) 5.5 or 0.5

7. (a) $p = 16,\ q = 31$ (c) (i) 7.9 (ii) 3.3 or – 1.8

8. – 8, 92 (b) – 12, $x = 3.5$ (c) (i) 6 (ii) 2.2 or 4.9

9. (a) $h = 1.5,\ k = 0$ (c) (i) 1.30 (ii) 0.17 (d) 0.31 or 0.86

10. (a) $h = -19,\ k = -12$ (c) (i) 8.5 (ii) 0.5
(d) (i) – 2 or 3.3 (ii) – 0.8 or 2.1

11. (a) $h = 13.3,\ k = 17.1$ (c) (i) 13.6 (ii) 1.7, 3.8

12. (a) $p = -7,\ q = -8,\ r = -11.5$ (b) (i) – 11.5 (ii) – 7.3
(c) $y = -6.8,\ x = 4.5$

13. (a) $h = 6.5,\ k = 6.05$ (c) (i) 7.04 (ii) 6.08 (d) (i) 2.6 or 6.2 (ii) 1.3

14. (a) P (–3, 0) Q (0, 12) R (4, 0)
(b) x = $\frac{1}{2}$ (c) y = $12\frac{1}{4}$ (d) (i) y = –3 (ii) y = x + 7

15. (1.5, 8.5)

16. h = $\frac{12}{x^2}$ (a) h = 72, k = 100.9
(c) (i) 1.2 or 3.2 (ii) 91.4 (iii) 3

17. (a) h = 20 (c) x = 3.5
(d) $1 \leq x \leq 4$ (e) 5.5 or –0.5

18. (a) h = 9.62 (d) 0.6
(e) x = 2.8 or 6.1 (f) $3.5 < x < 6.2$

19. (a) p = ll.9 q = 11.6 (c) 4.8
(d) l.4 or 2.7 (e) y = 12 x = 2.5

20. (b) x = 2 (c) h = 2.6, k = –3.6
(e) p = l, q = –4 (f) –3

21. (a) h = 0, k = –4 (b) –2.7, 0.7
(d) x < –3.3 or x > 2.3 (e) 3

22. 0, 0.45, 1.18 (a) (i) –0.2 (ii) 2.6
(b) 2.5 (c) 1.24

23. 8.5, 7 (a) (i) 6.7 (ii) 2.4 < x < 5
(b) 3.3 (c) 1.55 or 6.45

24. (a) h = 3.5, k= 3.5 (c) 8.2, 4.5
(d) –1.6 (f) 2.5 ≤ x ≤ 7.0

25. (a) p = –0.6 q = 0 (c) 1.6
(d) x = ± 2.55 (e) –2.1, –0.2, 2.3

26. (b) 3 (c) (i) 2.4 or –0.4 (ii) 3.6 or –0.6 (iii) –1.8 or 2.8
(d) –0.3 ≤ x ≤ 3.3

27. (a) h = –2, k = 4.5 (c) 1
(d) (i) x < l.4 (ii) 1 < x < 2 (e) 1.5

28. (a) –3.75, 0, l.2, 0 (c) –1.2, 0.55
(d) –l. 1 ≤ x ≤ 1 (e) 1.9

29. (a) 16.3, 5.5 (c) (i) 1.7 (ii) 0.4
(d) (1.35, 11.4) (e) –1.4

30. –13, – 4, 2, 1, – 4, –8 (a) 2, 1.3, –6.3 (b) 1.87 or –0.37 ; 2.5 or –1

31. 15, 35 (b) (i) 13.9, 7.2 (ii) –3.3, –0.4, 1.7 ; –3.6

32. (a) $p = 0.13$, $q = 0.06$ (c) (i) 2.2 , 4.7 (ii) 6.06 , 0.62

33.8, 2, 2, 6.6 (a) 1.2 (b) 5.5 or 2.7 (c) –2.7

34.–0.25 , –0.6 , 0.23 (a) 1.22 or –1.22 (b) $–1.54 < x < 1.79$ (c) –0.94

35.120, 128, 105, 72 (a) 1.4, 5.5 (b) $1.8 < x < 4.8$ (c) –23

36.(a) $p = 4.58$, $q = 8.05$, $r = 13.82$ (c) 3.5
(d) (i) 0.25, 2.7(ii) 0.53, 2.81 (e) $x < 0.34$ or $x > 2.26$

37. (a) $h = 3.7$, $k = 1.11$ (c) (i) 4.1 (ii) 6.4 (d) 0.2

Chapter 2

Secondary 4 Mathematics
Chapter 2 Further Graphs and Graphs Applied to Kinematics

ANSWERS FOR ENRICHMENT ACTIVITIES

Just For Fun (pg 42)

(a) 68 (multiply opposite number by 4)
(b) 35 (add the 6 digits)
(c) 65 ($0^3 + 1$, $1^3 + 1$, $2^3 + 1$, $3^3 + 1$, $\therefore 4^3 + 1 = 65$)
(d) 50 (add 3, add 6, add 12, $\therefore 24$ gives 50)
(e) 28 (1st line – 2 line) × 2

Just For Fun (pg 43)

No , Mr Tan.

Just For Fun (pg 45)

The car journey will have a lot of acceleration, deceleration and stops, while the train journey will accelerate, move with a fairly constant speed before decelerating to a stop at Johor Bahru.

Just For Fun (pg 58)

It is impossible for the car to achieve an average speed of 60 km/h for the whole journey no matter how fast it moves down the slope.

Just For Fun (pg 68)

1.

	(1)	*DC*	$mR \rightarrow$	
	(2)	*DC*	$\leftarrow m$	*R*
	(3)	*C*	$mD \rightarrow$	*R*
	(4)	*C*	$\leftarrow mR$	*D*
	(5)	*R*	$mC \rightarrow$	*D*
	(6)	*R*	$\leftarrow m$	*DC*
	(7)		$mR \rightarrow$	*DC*

Are there other possible ways of achieving this result?

Just For Fun (pg 70)

15, 55

Secondary 4 Mathematics
Chapter 2 Further Graphs & Graphs Applied to Kinematics

GENERAL NOTES

Revise the formula: speed = $\frac{\text{distance}}{\text{time}}$ as many pupils tend to forget this simple yet important formula. Distance moved for t hours at v km/h is tv km. Time taken to travel d km at v km/h is $\frac{d}{v}$ h.

The concept of tangency and gradient is new to pupils who are doing only elementary mathematics. Using Graphmatica to draw tangent to the graph will help pupils to visualize the concept better.

Common Errors

Pupils often think that 1 hour 15 minutes is 1.15 h and not 1.25 h. Pupils are also prone to making careless errors where different units are given in a problem, for example when speed is given as v km/h and time as t minutes, etc.

To avoid pupils giving ridiculous answers, teachers should ask them to think of real-life situations, for example, a motorist cannot possibly be moving at a speed of 500 km/h! When a motorist increases his speed, the time taken to cover the same distance will be shorter, etc.

Ask pupils to estimate the speed of a F1 car in a race. Ask them if they know the speed limits for cars on an Expressway, on a normal road with dual carriageway and for roads with single carriageway. Do they know the speed limits for goods vehicles?

NE MESSAGES

We must ourselves defend Singapore.

Page 50 Exercise 2a Q3 and Q5

The Singapore Navy purchased several Sjoormen submarines from Sweden in 1998. 23 crewmen are needed to man each submarine. Singapore is a small country, however we have to defend the country ourselves. Acquiring high-tech weapons and machines to defend the territorial waters and to protect the sea-lanes is vital to our economic survival and is a necessary part of our defence strategy. We cannot depend on other countries to defend us, we alone must defend Singapore.

The BIONIX Infantry Fighting Vehicle is a locally produced fighting machine. It is developed to suit our needs. It has proven to be a much better vehicle than the imported ones during the exercises conducted recently in Australia. The vehicle is equipped with 25 mm Bushmaster guns and three 7.62 mm automatic rifles. Each vehicle is manned by 10 soldiers.

Page 68 Review Questions 2 Q4

The F-16 fighter plane is one of the world's most advanced fighter jets. It is very sophisticated and one of the most expensive fighter planes in the world. Singapore has spared no efforts to acquire the most modern and effective weapon systems to protect itself from many potential aggressors.

XYZ SECONDARY SCHOOL

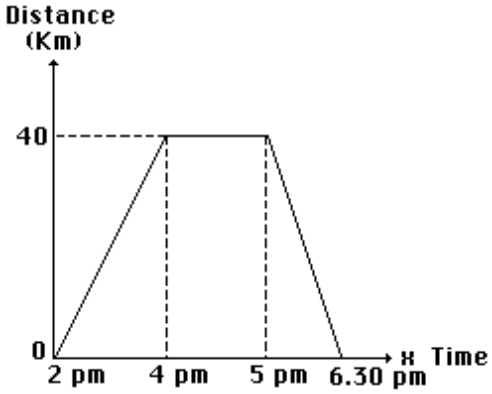
Name: _____ ( ) Date: _____

Time allowed: min

Class: _____ Marks: /7

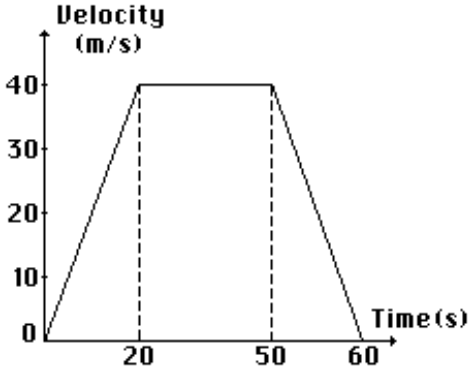
Secondary 4 Multiple-Choice Questions
Chapter 2 Further Graphs and Graphs Applied to Kinematics

1. The diagram represents a travel graph of a cyclist. Which of the following is/are true?
   I. The speed of the outward journey is less than the speed of the return journey.
   II. The cyclist rested for one hour during the journey.
   III. The total distance travelled is 95 km.
   (A) I only (B) II only
   (C) I and II only (D) II and III only
   (E) all of them ( )



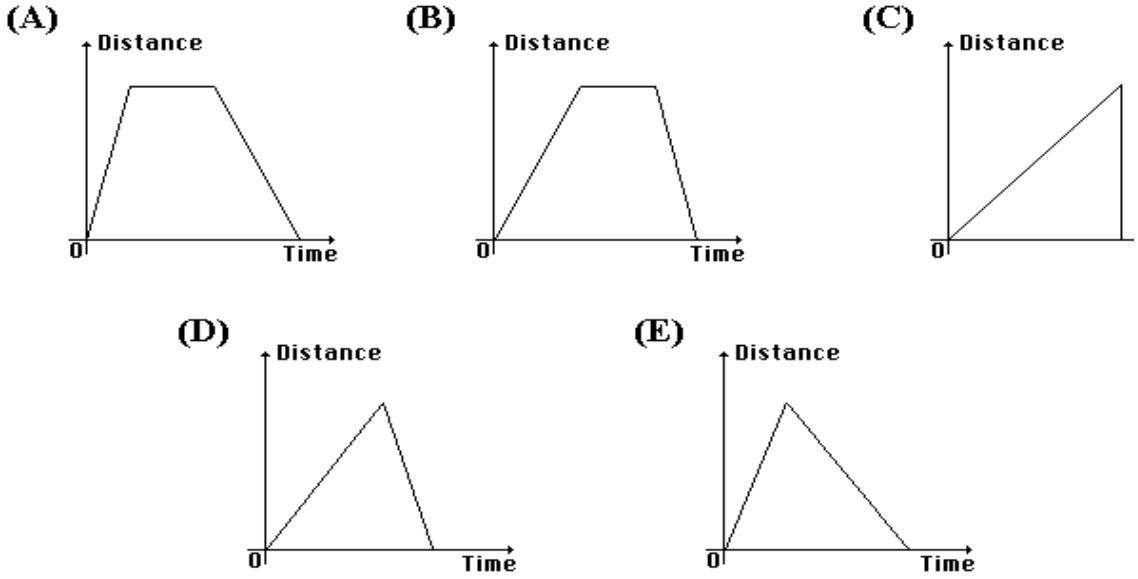
2. The figure shows the velocity-time graph of a fast train travelling from station *P* to station *Q*. Find the distance, in metres, between stations *P* and *Q*.

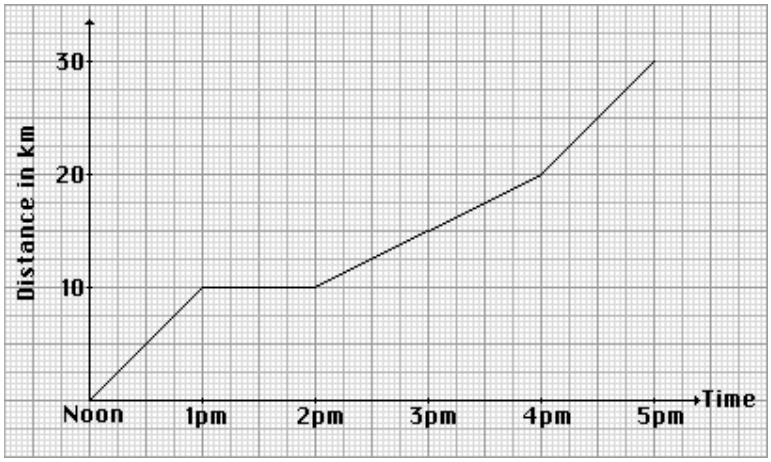
   (A) 1 200 (B) 1 600
   (C) 1 800 (D) 2 000
   (E) 2 400 ( )



3. Refer to the figure in Question 2, the acceleration of the first train during the first 10 seconds of its motion is
   (A) 8 m/s^2 (B) 4 m/s^2 (C) 2 m/s^2 (D) 1 m/s^2
   (E) 0.5 m/s^2 ( )

4. A man travels from Shenton Way to Bishan at a constant speed. After he reaches Bishan, he returns immediately but at a greater speed. Which of the following distance-time graphs is correct? ( )

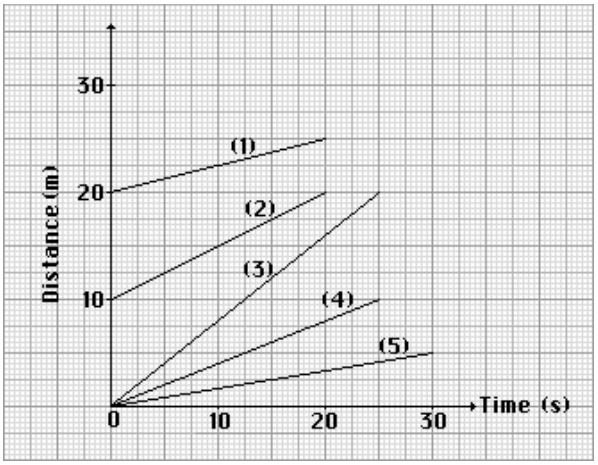


5. 

The diagram shows the travel graph of Mr Ong travelling from Jurong to visit Mr Chan in Changi. At the same time, Mr Tay starts from Changi to visit his friend Mr Lee in Jurong. Mr Tay travels from Changi to Jurong at a constant speed of 7.5 km/h. When will Mr Ong and Mr Tay meet?

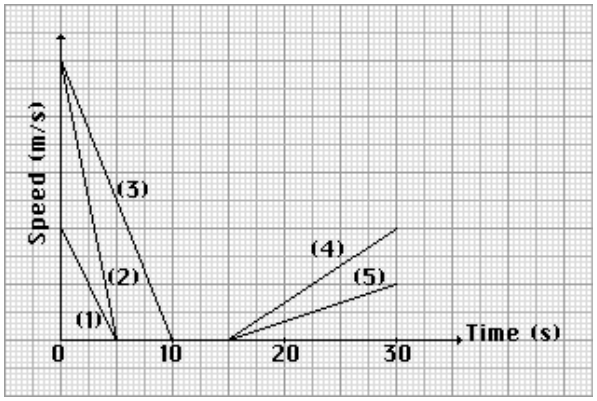
(A) 2.06 pm (B) 2.15 pm (C) 2.24 pm (D) 2.45 pm
(E) 4 pm ( )

6. The diagram shows 5 travel graphs, (1), (2), (3), (4) and (5). Which one is travelling at the lowest speed?



(A) (1) (B) (2)
(C) (3) (D) (4)
(E) (5) ( )

7. The diagram shows the speed-time graphs of 5 motorists. Which one has the greatest acceleration?



(A) (1) (B) (2)
(C) (3) (D) (4)
(E) (5) ( )

Answers

1. C
2. C
3. D
4. D
5. C
6. E
7. D

XYZ SECONDARY SCHOOL

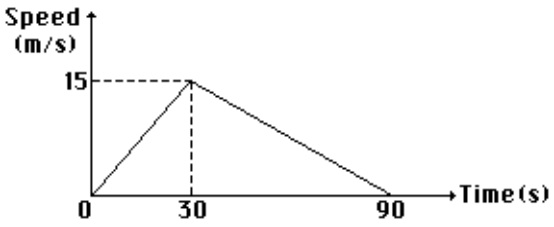
Name: _____ ( ) Date: _____

Time allowed: min

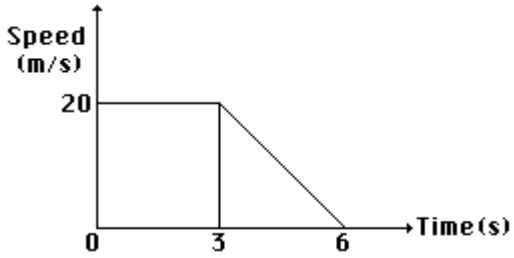
Class: _____ Marks:

Secondary 4 Mathematics Test
Chapter 2 Further Graphs and Graphs Applied to Kinematics

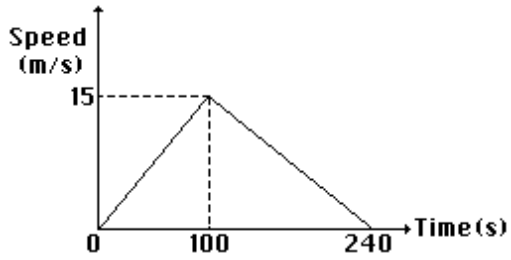
1. The diagram is the speed-time graph of a bus starting from rest. The bus accelerated at a constant rate for 30 seconds, reaching a speed of 15 m/s. The driver then braked so that the bus came to rest in a further 60 seconds. Calculate
   (a) the acceleration of the bus during the first 30 seconds, [2]
   (b) the total distance travelled during the 90 seconds, [2]
   (c) the average speed during the 90 seconds. [2]



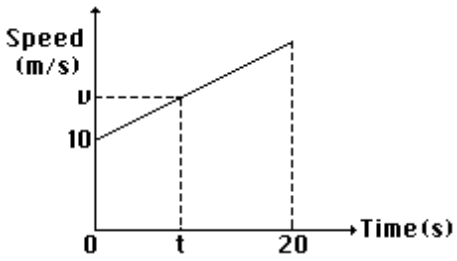
2. The diagram shows the speed-time graph of a particle during a period of 6 seconds. Calculate
   (a) the distance travelled in the first 3 seconds, [2]
   (b) the deceleration of the particle during the last 3 seconds, [2]
   (c) the average speed of the particle during the 6 seconds. [2]



3. The diagram shows the speed-time graph of a tube train over a period of 240 seconds. Calculate
   (a) the acceleration of the train during the first 100 seconds, [2]
   (b) the average speed of the train during the 240 seconds. [2]



4. The diagram is the speed-time graph of a train which accelerates at 1.5 m/s^2 for 20 seconds. The initial speed of the train is 10m/s.

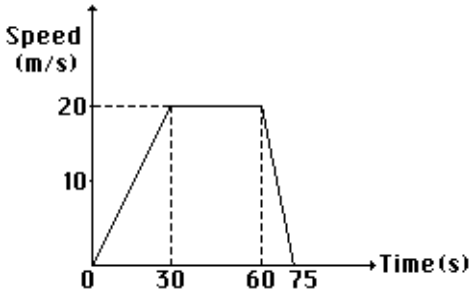


(a) Calculate the speed of the train at the end of the 20 seconds. [2]

(b) Calculate the distance travelled during the first 10 seconds. [2]

(c) Given that the speed after t seconds is v m/s, express v in terms of t. [2]

5. The diagram shows the speed-time graph of a car. Calculate



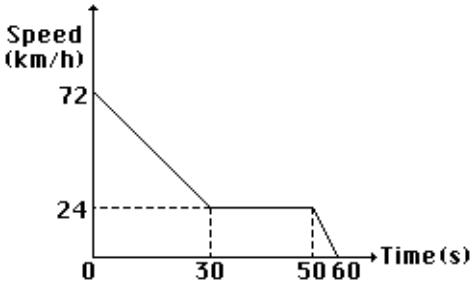
(a) the acceleration of the car during the first 30 seconds, [2]

(b) the total distance the car travels from rest before it begins to decelerate, [2]

(c) the deceleration of the car during the last 5 seconds of its motion. [2]

(d) Convert 20 m/s into kilometres per hour. [1]

6. The diagram shows the speed-time graph of a train.



Calculate

(a) the deceleration of the train during the first 30 seconds, giving your answer in m/s^2, [2]

(b) the total distance covered in the first 60 seconds, [2]

(c) the average speed of the train, giving your answer in m/s. [2]

7. A car passes a checkpoint with an initial speed of 6 m/s. It then accelerates uniformly for 5 seconds at 0.8 m/s^2. It maintains its speed at this constant rate for a further 12 seconds before it decelerates uniformly to a stop in a further 8 seconds.

(a) Draw the speed-time graph for the car. [2]

(b) Calculate the total distance travelled. [2]

(c) Calculate the average speed for the journey. [2]

8. A motorist starting from rest accelerates uniformly to a maximum speed of 30 m/s which he then maintains for the next 4 minutes. He then applies his brakes and decelerates to rest at a rate numerically equal to twice his previous acceleration. Sketch a velocity-time graph using the above information.

Given that the total distance travelled is 7.875 km, calculate

(a) the total time taken to complete the journey, [2]

(b) the acceleration of the motorist, [2]

(c) the time needed to travel the first 5 km of the journey. [2]

9. A bus, travelling at 15 m/s passes a stationary car. Immediately it accelerates at 0.5 m/s for a further 10 seconds and then continues at a constant speed. Thirty seconds after the bus has passed, the car starts from rest with an acceleration of $1 m/s^2$ for a period of 30 seconds. It then maintains this speed.
Using a scale of 1 cm to represent 5 seconds on the horizontal axis and 1 cm to represent 5 m/s on the vertical axis, draw the speed-time graph for the above journey.
Hence calculate

(a) the difference of speed between the bus and the car $\frac{1}{4}$ minutes after the car started its journey, [2]
(b) the time when the speed of the car and the bus are equal, [2]
(c) the time taken for the car to overtake the bus. [2]

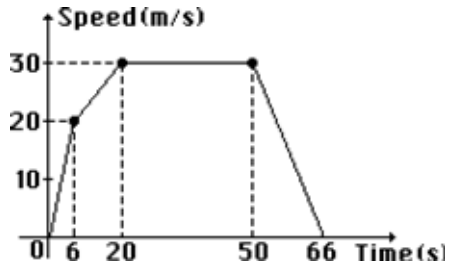
10. (a) A car moves at a constant speed of 54 km/h. Find, in metres, the distance travelled by the car in 12 seconds. [1]

(b) A man cycles for 3 hours at $12\frac{1}{3}$ km/h and then walks for 2 hours at 4 km/h. Calculate the man's average speed over the whole journey. [1]

11. The speed-time graph of a car consists of the straight lines shown in the diagram. Find

(a) the acceleration of the car during the first 6 seconds, [1]

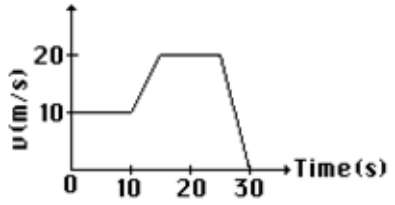
(b) the total distance travelled in the 66 seconds. [2]



12. A train travels at an average speed of 60 km/h for 2 hours and then increases its average speed to 90 km/h for the next hour. Find its average speed for the three hour journey. [2]

13. The figure shows the motion of a particle over a time of 30 seconds.

(a) What was the initial velocity? [1]
(b) Find the distance travelled when the particle was moving with constant velocity. [2]
(c) Find the average speed over the period of 30 seconds. [2]

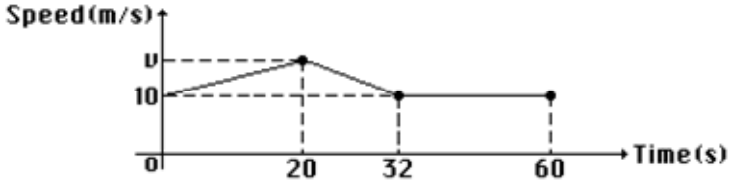


14. The diagram shows the speed-time graph of a car during a period of 60 seconds. The distance travelled in the first 20 seconds is 250 m. Calculate

(a) the maximum speed v, [1]

(b) the retardation during the motion, [1]

(c) the average speed during the 60 seconds. [3]

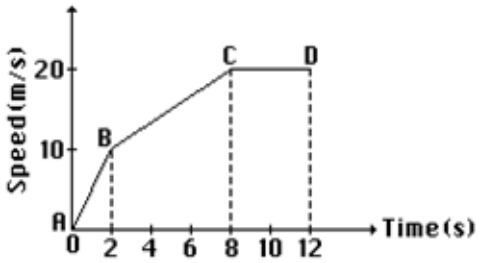


15. The graph shows the speed of a particle during a period of 12 seconds. Find

(a) the total distance travelled, [2]

(b) the average speed during the 12 seconds, [1]

(c) the rate of change of speed from the 2nd to the 8th second. [1]



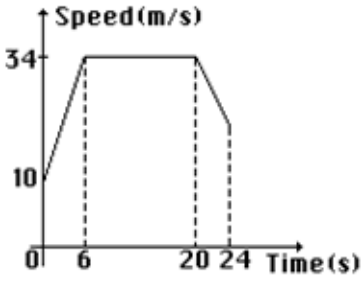
16. A cyclist travels at 30 km/h for 50 % of a journey, 25 km/h for 30 % of the journey and 15 km/h for the remaining journey. Calculate the average speed, to the nearest km/h, of the cyclist for the whole journey. [3]

17. The diagram is the speed-time graph of an object during a period of 24 seconds.

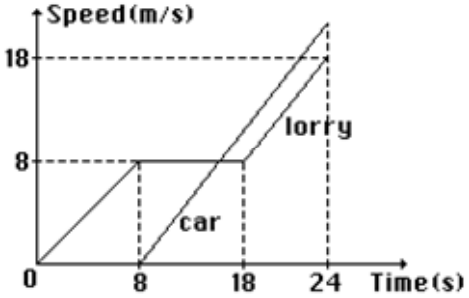
(a) Calculate the speed of the object when $t = 4$ sec. [2]

(b) Calculate the total distance moved in the first 20 seconds. [2]

(c) Given that the acceleration of the object between $t = 20$ sec and $t = 24$ sec is -4.5 m/s^2, calculate the speed of the object when $t = 24$ sec. [2]



18. The diagram shows the speed-time graphs of a lorry and a car during the first 24 seconds. Calculate

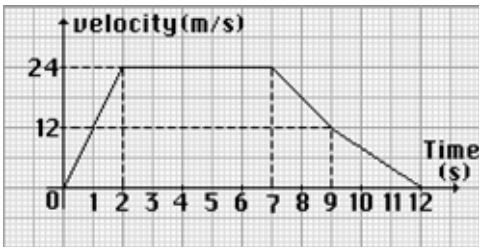


(a) the acceleration of the lorry during the first 8 seconds, [1]

(b) the average speed of the lorry during the first 18 seconds of its motion, [2]

(c) the speed of the car at t = 24 sec, given that the car overtakes the lorry at time t = 24 sec. [2]

19. The diagram shows the velocity-time graph of a particle. Find



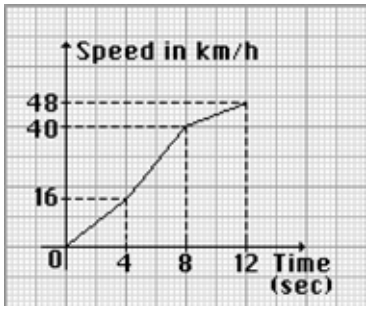
(a) the acceleration of the particle, [1]

(b) the greatest retardation of the particle, [1]

(c) the distance of the particle at time t = 10 seconds, [1]

(d) the distance travelled in the last 5 seconds of its motion. [2]

20. The diagram shows the speed-time graph of a vehicle over a period of 12 seconds. Calculate

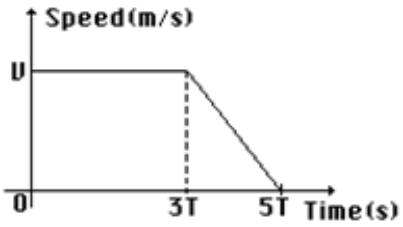


(a) the speed when t = 10 seconds, [2]

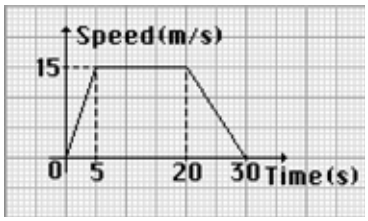
(b) the value of t when the speed is 34 km/h, [2]

(c) the distance moved, in metres, from t = 4 to t = 8 seconds. [2]

21. The diagram shows the speed-time graph of a particle over a period of 5T seconds. Given that the average speed for the whole journey is 24 m/s and that the retardation is $1\frac{7}{8}$ m/s^2, calculate the value of V and of T. [4]



22. The diagram shows the speed-time graph of a particle over a period of 30 seconds.



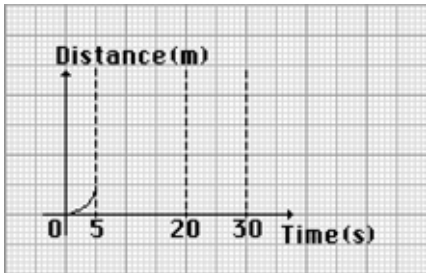
(a) Calculate

(i) the acceleration of the particle, [1]

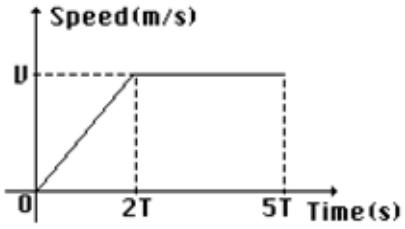
(ii) the speed of the particle at $t = 24$ sec, [1]

(iii) the average speed of the particle during the 30 seconds. [3]

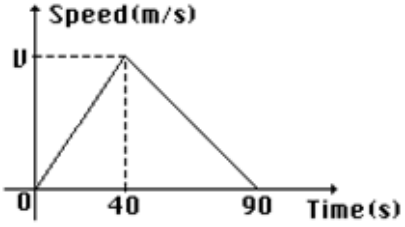
(b) Sketch the distance-graph for the journey on the graph given. [2]



23. The diagram is the speed-time graph of a particle. Given that the acceleration is 3 m/s^2 for the first $2T$ seconds and the total distance travelled in $5T$ seconds is 0.5 km, find the value of T and of V. [4]



24. The diagram shows the speed-time graph of a particle over a period of 90 seconds. Calculate

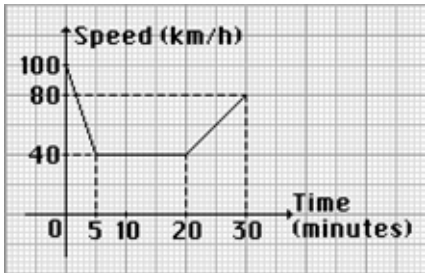


(a) the value of V if the total distance travelled is 1.08 km, [2]

(b) the total distance travelled in the first 60 seconds, [2]

(c) the deceleration in m/s^2. [1]

25. The diagram shows the speed of a car in km/h over a period of 30 minutes. Find



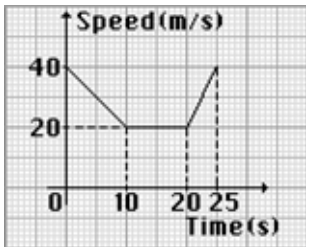
(a) the distance in kilometres travelled at constant speed, [2]

(b) the total distance travelled, [2]

(c) the average speed in km/h during the 30 minutes, [2]

(d) the acceleration in km/h^2. [2]

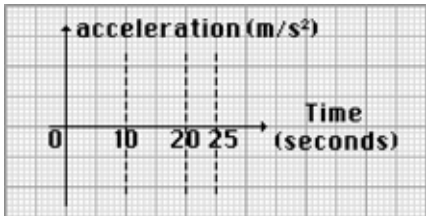
26. The diagram shows the speed-time graph for a particle over a period of 25 seconds.



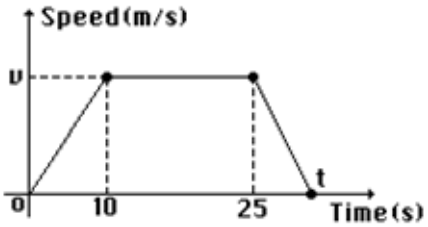
(a) Find the retardation of the particle during the first 10 seconds of its motion. [1]

(b) Find the average speed of the particle during the 25 seconds of its motion. [3]

(c) Sketch the acceleration-time graph of the car for the whole 25 seconds of its motion on the axes provided. [2]



27. The diagram shows the speed-time graph of a car which accelerated uniformly from rest at a rate of 2 m/s^2 for 10 seconds to reach a speed of v m/s. The car then continues at this speed for another 15 seconds before the brakes are applied. Given that the rate of acceleration is half the rate of deceleration, calculate



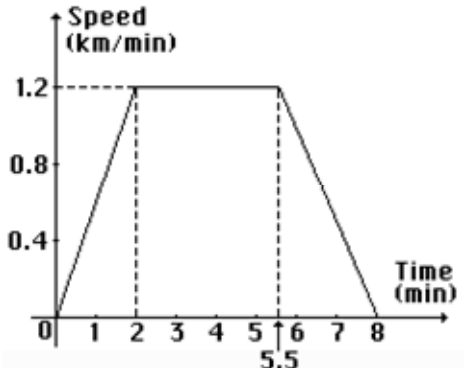
(a) the value of v, [1]

(b) the distance travelled during the first 20 seconds, [2]

(c) the distance travelled during deceleration, [1]

(d) the average speed for the whole journey. [1]

28. The diagram shows the speed-time graph for a train moving from Town *A* to Town *B*.



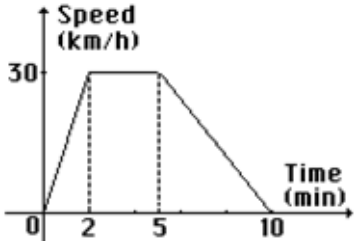
(a) Find the distance of *AB* in km. [2]

(b) Find the acceleration of the train in m/s^2. [3]

(c) Find the speed of the train at the time $t = 6$ min. [1]

(d) A car, travelling at an average speed of x km/min, leaves Town *A* by the same route as the train, 1 minute later, and it takes the car 4 minutes to overtake the train. Find the value of x and the distance from Town *A* where the overtaking takes place. [3]

29. The diagram shows the speed-time graph of a vehicle over a period of 10 minutes. Calculate



(a) the acceleration of the vehicle during the first 2 minutes in km/h^2, [2]

(b) the average speed during the 10-minute period, [2]

(c) the speed of the vehicle at time $t = 8$ minutes. [2]

30. The speed, v m/s, of a car, t seconds after passing a stationary motorcyclist is given in the following table.

t (s)	0	1	2	3	4	5	6	7
v (m/s)	5	11	15	17	17	15	11	5

(a) Using a scale of 2 cm to represent 1 second on the horizontal axis and a scale of 1 cm to represent 1 m/s on the vertical axis, draw the speed-time graph of the car. [2]

(b) Estimate from your graph

(i) the acceleration of the car $2\frac{1}{2}$ seconds after passing the stationary motorcyclist, [1]

(ii) the times at which the car has a speed of 14 m/s. [2]

(c) The stationary motorcyclist, intending to pursue the car, starts his engine 2 seconds after the car has passed him. Assuming that he has a constant acceleration of $4\frac{1}{2}$ m/s^2, add to your graph the speed-time graph of the motorcyclist, and find the time at which the car and the motorcyclist have the same speed. [2]

Answers

1. (a) $0.5\ \text{m/s}^2$ (b) 675 m (c) 7.5 m/s

2. (a) 60 m (b) $6\frac{2}{3}\ \text{m/s}^2$ (c) 15 m/s

3. (a) $0.15\ \text{m/s}^2$ (b) 7.5 m/s

4. (a) 40 m/s (b) 175 m (c) $v = 10 + 1.5t$

5. (a) $\frac{2}{3}\ \text{m/s}^2$ (b) 900 m (c) $1\frac{1}{3}\ \text{m/s}^2$ (d) 72 km/h

6. (a) $\frac{4}{9}\ \text{m/s}^2$ (b) $566\frac{2}{3}$ m (c) $9\frac{4}{9}$ m/s

7. (b) 200 m (c) 8 m/s

8. (a) 285 s (b) $1\ \text{m/s}^2$ (c) $181\frac{2}{3}$ s

9. (a) 5 m/s (b) 20 seconds after car started journey
(c) 102.5 seconds after car started journey

10. (a) 180 m (b) 9 km/h

11. (a) $3\frac{1}{3}\ \text{m/s}^2$ (b) 1550 m

12. 70 km/h

13. (a) 10 m/s (b) 300 m (c) $14\frac{1}{6}$ m/s

14. (a) 15 (b) $\frac{5}{12}\ \text{m/s}^2$ (c) $11\frac{1}{3}$ m/s

15. (a) 180 m (b) 15 m/s (c) $1\frac{2}{3}\ \text{m/s}^2$

16. 24 km/h

17. (a) 26 m/s (b) 608 m (c) 16 m/s

18. (a) $1\ \text{m/s}^2$ (b) $6\frac{2}{9}$ m/s (c) $23\frac{3}{4}$ m/s

19. (a) $12\ \text{m/s}^2$ (b) $6\ \text{m/s}^2$ (c) 9 m/s (d) 54 m

20. (a) 44 km/h (b) 7 sec (c) $31\frac{1}{9}$ m

21. $V = 30,\ T = 8$

22. (a) (i) $3\ \text{m/s}^2$ (ii) 9 m/s (iii) 11.25 m/s

23. $V = 30,\ T = 5$

24. (a) $V = 24$ (b) 864 m (c) $0.48\ \text{m/s}^2$

25. (a) 10 km (b) $25\frac{5}{6}$ km (c) $51\frac{2}{3}$ km/h (d) $240\ \text{km/h}^2$

26. (a) $2\ \text{m/s}^2$ (b) 26 m/s

27. (a) 20 m/s (b) 300 m (c) 50 m (d) 15 m/s

28. (a) 0.115 km (b) $\frac{1}{6}\ \text{m/s}^2$ (c) 0.96 km/min (d) $x = 1.2$, 6 km

29. (a) $900\ \text{km/h}^2$ (b) 19.5 km/h (c) 12 km/h

30. (b) (i) $2\ \text{m/s}^2$ (ii) 1.7 s or 5.3 s (c) 5.2 sec

Chapter 3

Secondary 4 Mathematics
Chapter 3 Vectors in Two Dimensions

ANSWERS FOR ENRICHMENT ACTIVITIES

Just For Fun (pg 79)
Route A.

Secondary 4 Mathematics
Chapter 3 Vectors in Two Dimensions

GENERAL NOTES

You may start the topic by asking the following questions:

1. Will a ship sailing 8 km on a bearing of 075° get to the same place as one sailing 8 km on a bearing of 165°?

2. Will an aeroplane going at 750 km/h north get to the same place as one going at 750 km/h east?

3. If a man pushes a box with a force of 50 newtons from the top of the box, is he doing the same thing as pushing it with a force of 50 newtons from the side of the box?

Discussion of the answers should help pupils realise that displacement, velocity and force are all vectors.

To help pupils understand the significance of vectors, you may ask them to imagine a boat drifting aimlessly in an ocean with current flowing due north and a wind blowing from the east. To simplify the problem, ask them to imagine that there are no waves in the ocean and that the motion of the boat depends solely on the current and the wind. Ask them how the boat will move. Will it move due north with the current, due west with the wind, or somewhere in between? Help them see that the exact direction will depend on the strength and direction of the current and the wind, and not on the exact location of the boat. Vectors can be used to solve the problem here. They are needed to represent the speed and the direction of the current and the wind. The two vectors are combined to obtain the resultant vector that will represent the boat's drift. Let the pupils discuss how the two vectors can be combined.

At this level, pupils are taught to understand that a vector possesses both magnitude and direction. You may want to point out to them that not all vectors possess these attributes. Some of the pupils may have to deal with such vectors at Pre-U level. In the study of **Linear Algebra**, the essential things to note about vectors are the laws of combination which they obey.

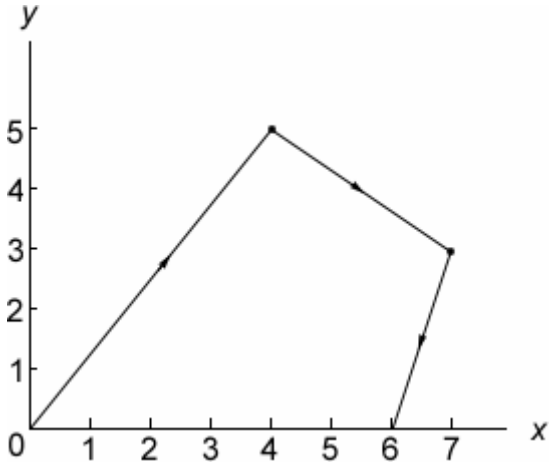
To make your discussion more interesting, you can ask the pupils to consider stamp collections. Are stamp collections vectors? Give them an example. Suppose a boy collects only Singapore and Malaysia stamps. A record of his stamp collection is shown by the table below:

	1st July	8th July	15th July	22nd July	29th July
Singapore	0	Bought 4	bought 3	bought 0	lost 1
Malaysia	0	Bought 5	sold 2	bought 0	sold 3

To find the total number of stamps of each country, the boy can write

$$\begin{pmatrix}0\\0\end{pmatrix}+\begin{pmatrix}4\\5\end{pmatrix}+\begin{pmatrix}3\\-2\end{pmatrix}+\begin{pmatrix}0\\0\end{pmatrix}+\begin{pmatrix}-1\\-3\end{pmatrix}=\begin{pmatrix}6\\0\end{pmatrix},$$

and then he can draw a diagram shown below.



XYZ SECONDARY SCHOOL

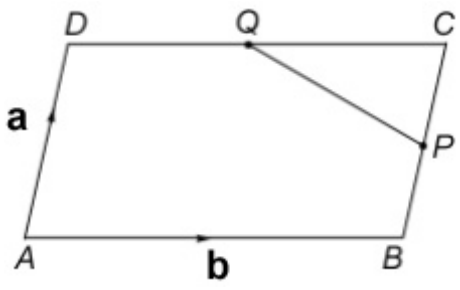
Name: _____ ( ) Date: _____

Time allowed: min

Class: _____ Marks: /8

Secondary 4 Multiple-Choice Questions
Chapter 3 Vectors in Two Dimensions

1. In the diagram, *ABCD* is a parallelogram, and *P* and *Q* are the midpoints of *BC* and DC respectively. If $\overrightarrow{AB}$ = **a** and $\overrightarrow{AD}$ = **b**, then $\overrightarrow{PQ}$ = ___.



(A) $\frac{1}{2}$(**a**–**b**) (B) $\frac{1}{2}$(**b**–**a**) (C) **a**+**b**
(D) $\frac{3}{2}$(**a**+**b**) (E) $\frac{3}{2}$(**b**–**a**) ( )

2. If **u** = $\begin{pmatrix}-3\\-2\end{pmatrix}$, **v** = $\begin{pmatrix}-5\\4\end{pmatrix}$, and **w** = $\begin{pmatrix}9\\-16\end{pmatrix}$, then **w** =
(A) 2**u**–3**v** (B) 3**u**–2**v** (C) –3**u**+2**v** (D) –2**u**–3**v** (E) –2**u**+3**v** ( )

3. A triangle *LMN* has vertices at *L*(3, –2), *M*(10, 2) and *N*(4, 8). *P* is a point on *MN* such that $\overrightarrow{PN} = 2\overrightarrow{MP}$. Calculate $\overrightarrow{LP}$.

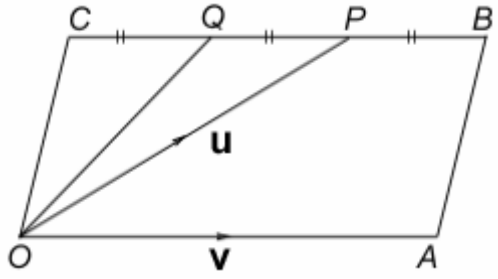
(A) $\begin{pmatrix}5\\6\end{pmatrix}$ (B) $\begin{pmatrix}6\\5\end{pmatrix}$ (C) $\begin{pmatrix}3\\8\end{pmatrix}$ (D) $\begin{pmatrix}8\\3\end{pmatrix}$ (E) $\begin{pmatrix}5\\3\end{pmatrix}$ ( )

4. The coordinates of *A*, *B* and *C* are (–1, 4), (1, 0) and (4, 5) respectively. The value of $\left|\overrightarrow{AB} + \overrightarrow{BC}\right|$ is
(A) $\sqrt{20} + \sqrt{34}$ (B) 10 (C) $\sqrt{10}$ (D) $\sqrt{26}$ (E) $\sqrt{54}$ ( )

5. $\overrightarrow{PQ} = \begin{pmatrix} -8 \\ 20 \end{pmatrix}$ and $\overrightarrow{RS} = \begin{pmatrix} h \\ -25 \end{pmatrix}$. Given that $\overrightarrow{RS}$ is parallel to $\overrightarrow{PQ}$, find the value of *h*.

(A) –62.5 (B) –10 (C) 3 (D) 10 (E) 62.5 ( )

6. In the diagram, *OABC* is a parallelogram, and *P* and *Q* are points on *BC* such that *BP* = *PQ* = *QC*. Given that $\overrightarrow{OP}$ = **u** and that $\overrightarrow{OA}$ = **v**, express the vector $\overrightarrow{OQ}$ in terms of **u** and **v**.



(A) $\frac{1}{3}(\mathbf{u} - \mathbf{v})$ (B) $\frac{1}{3}(\mathbf{u} + \mathbf{v})$

(C) $\frac{1}{3}(3\mathbf{u} - \mathbf{v})$ (D) $\frac{1}{3}(3\mathbf{u} + \mathbf{v})$

(E) $\frac{1}{3}(\mathbf{v} - \mathbf{u})$ ( )

7. *ABCDEF* is a regular hexagon. Let **a, b,** and **c** be the position vectors of *A*, *B* and *C* respectively. The vector $\overrightarrow{CD}$ = _____ .

(A) **a**+2**b**+**c** (B) **a**–2**b**+**c** (C) **a**+2**b**–**c** (D) **a**–2**b**–**c** (E) **a**+2**b**–3**c** ( )

8. Using the data given in Question 7, the vector $\overrightarrow{OE}$ = _____ .

(A) –**b**+2**c** (B) **b**–2**c** (C) 2**a**+**b**+2**c** (D) 2**a**+**b**–2**c** (E) **2a**–3**b**+2**c** ( )

Answers

1. B	2. A	3. A	4. D
5. D	6. C	7. B	8. E

XYZ SECONDARY SCHOOL

Name: _____ ( ) Date: _____

Time allowed: min

Class: _____ Marks:

Secondary 4 Mathematics Test
Chapter 3 Vectors in Two Dimensions

1. *PQRS* is a parallelogram. Given that $\overrightarrow{PQ} =$ and $\overrightarrow{PR} = \begin{pmatrix} 8 \\ -6 \end{pmatrix}$,

(a) Find $\left|\overrightarrow{PR}\right|$. [1]

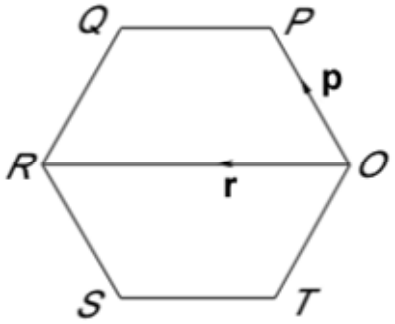
(b) Express, as column vectors,

(i) $\overrightarrow{QR}$, [1] (ii) $\overrightarrow{SQ}$. [1]

(c) The point *A* lies on *SQ* produced such that $4\overrightarrow{QA} = \overrightarrow{SQ}$. Express $\overrightarrow{PA}$ as a column vector. [2]

2. Given that $\overrightarrow{PQ} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$, $\overrightarrow{QR} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$, and $\overrightarrow{RS} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$, find $\overrightarrow{SP}$. [2]

3. In the diagram, *OPQRST* is a regular hexagon, and $\overrightarrow{OP}$ = **p** and $\overrightarrow{OR}$ = **r**.



Express the following vectors, as simply as possible, in terms of **p** and/or **r**:

(a) $\overrightarrow{RS}$, [1]

(b) $\overrightarrow{ST}$, [1]

(c) $\overrightarrow{PR}$, [1]

(d) $\overrightarrow{TR}$. [2]

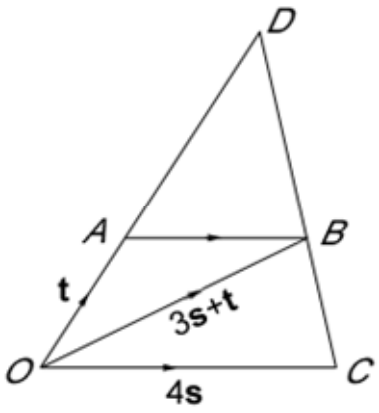
4. It is given that P is the point (2, 3), Q is the point (14, 11) and A is the point on PQ such that $\overrightarrow{AQ} = \frac{1}{3}\overrightarrow{PA}$. Express, as column vectors,

(a) $\overrightarrow{PQ}$, [1]

(b) $\overrightarrow{QA}$, [2]

(c) $\overrightarrow{PA}$. [1]

5. In the diagram, the lines AB and OC are parallel, OAD and CBD are straight lines.



It is given that $\overrightarrow{OC} = 4\mathbf{s}$, $\overrightarrow{OA} = \mathbf{t}$ and $\overrightarrow{OB} = 3\mathbf{s} + \mathbf{t}$.

(a) Express $\overrightarrow{AB}$ in terms of **s** and **t**. [1]

(b) Find the numerical values of

(i) $\frac{\text{area of } \Delta ABD}{\text{area of } ABCO}$, [2]

(ii) $\frac{\text{area of } \Delta OAB}{\text{area of } \Delta OBC}$. [2]

6. $\overrightarrow{AB} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$, $\overrightarrow{BC} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$, $\overrightarrow{CD} = \begin{pmatrix} 12.5 \\ \mathbf{t} \end{pmatrix}$

(a) Express $2\overrightarrow{AB} - 3\overrightarrow{BC}$ as a column vector. [1]

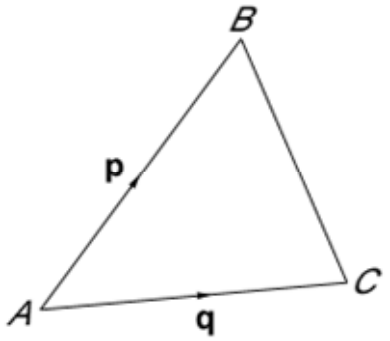
(b) Given that $\overrightarrow{CD}$ is parallel to $\overrightarrow{BC}$,

(i) find the value of **t,** [1]

(ii) find $\overrightarrow{DA}$. [1]

(c) Find $\left|\overrightarrow{AC}\right|$. [1]

7. In the diagram, $\overrightarrow{AB}$ = **p** and $\overrightarrow{AC}$ = **q**, *P* is the point on *BC* such that $\overrightarrow{CP} = \frac{1}{3}\overrightarrow{CB}$. Express the following, as simply as possible, in terms of **p** and/or **q**:



(a) $\overrightarrow{CB}$, [1]

(b) $\overrightarrow{AP}$. [2]

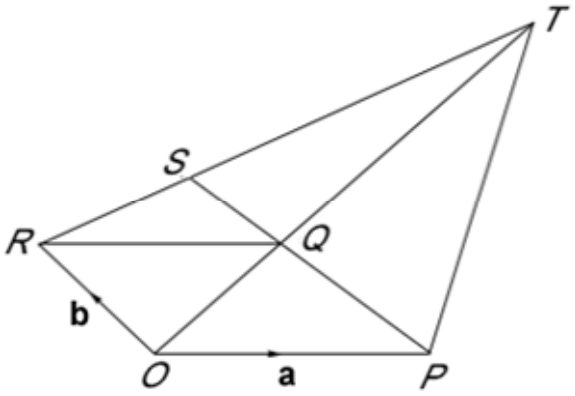
8. It is given that $\overrightarrow{PQ} = \begin{pmatrix} -12 \\ 5 \end{pmatrix}$.

(a) Calculate $|\overrightarrow{PQ}|$. [1]

(b) Given that *Q* is the point (4, –7), find the coordinates of the point *P*. [2]

(c) Given that $\overrightarrow{RS}$ is parallel to $\overrightarrow{PQ}$ and that $|\overrightarrow{RS}| = 2|\overrightarrow{PQ}|$, write down two possible column vectors representing $\overrightarrow{RS}$. [2]

9. *OPQR* is a parallelogram. *OQT, PQS* and *RST* are straight lines.



(a) Given that $\overrightarrow{OP}$ = **a**, $\overrightarrow{OR}$ = **b** and $\overrightarrow{QT}$ = 2(**a** + **b**), express the following, as simply as possible, in terms of **a** and **b**,

(i) $\overrightarrow{PT}$, [1]

(ii) $\overrightarrow{OT}$, [1]

(iii) $\overrightarrow{PR}$, [1]

(iv) $\overrightarrow{RT}$. [1]

(b) Given that $\frac{RS}{RT} = k$, express $\overrightarrow{PS}$ in terms of *k*, **a** and **b**. [3]

10. P is the point (1, 4), Q is the point (–9, 19) and X is the point on PQ such that $\frac{PX}{XQ} = \frac{1}{4}$.

(a) Express, as column vectors,

(i) $\overrightarrow{PQ}$, [1]

(ii) $\overrightarrow{PX}$. [1]

(b) Find the coordinates of X. [2]

11. (a) Given that $\overrightarrow{OP} = \begin{pmatrix} 6 \\ -8 \end{pmatrix}$, $\overrightarrow{OQ} = \begin{pmatrix} 2l \\ -3l \end{pmatrix}$ and $\overrightarrow{PQ} = \begin{pmatrix} m \\ 5 \end{pmatrix}$, find the values of l and m. [2]

(b) A point B lies on PQ such that $\overrightarrow{PB} = \begin{pmatrix} 4 \\ n \end{pmatrix}$. Find

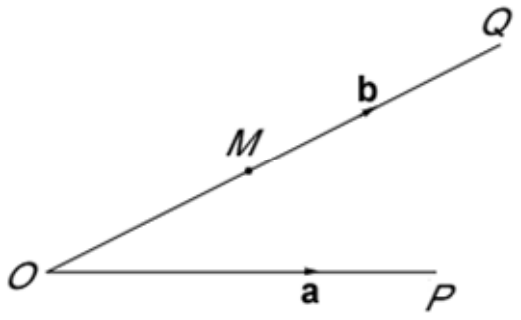
(i) the value of n, [1]

(ii) $\overrightarrow{OB}$. [1]

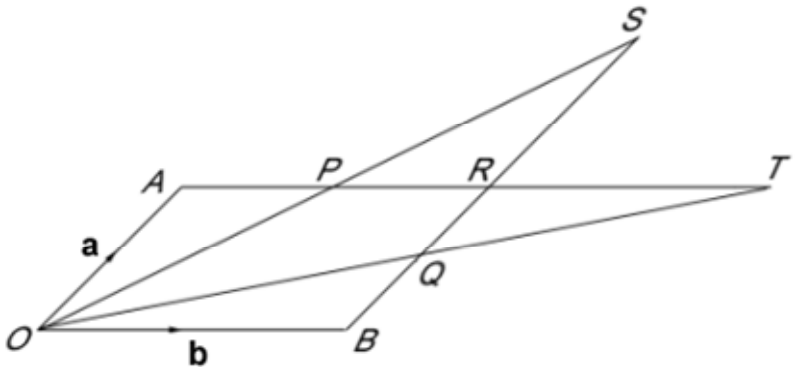
12. In the diagram, $\overrightarrow{OP}$ = **a** and $\overrightarrow{OQ}$ = **b**.

(a) Given that k (**a** + **b**) = 3**a** – (h + 1) **b**, where k and h are constants, find the values of k and h. [2]

(b) Given that M is the midpoint of OQ, express $\overrightarrow{PM}$ in terms of **a** and **b**. [2]



13. In the diagram, *OBRA* is a parallelogram. *P* is the midpoint of *AR* and *Q* is the midpoint of *BR*. The line *OP* produced meets *BR* produced at *S* and the line *OQ* produced meets the line *AR* produced at *T*. Given that $\overrightarrow{OA}$ = **a** and that $\overrightarrow{OB}$ = **b**,



(a) express, in terms of **a** and **b**,

(i) $\overrightarrow{OS}$, [2]

(ii) $\overrightarrow{OT}$, [2]

(iii) $\overrightarrow{ST}$, [1]

(iv) $\overrightarrow{AB}$, [1]

(b) deduce that *ST* is parallel to *AB*. [1]

14. *P* is the point (2, 1) and *Q* is the point (3, –2).

(a) The point *R* is such that its position vector with respect to the origin *O* is equal to $\overrightarrow{PQ}$. Find the coordinates of *R*. [2]

(b) *S* is the point (13, –4) and is such that $\overrightarrow{OS} = m\overrightarrow{OP} + n\overrightarrow{OQ}$, where *m* and *n* are constants. Find the values of *m* and *n*. [3]

15. Let the position vectors of two points *A* and *B* with respect to an origin *O* be **a** and **b** respectively. The points *C, D* and *E* are such that $\overrightarrow{OC}$ = **a** + **b**, $\overrightarrow{OD}$ = **a** + $\frac{1}{2}$**b** and $\overrightarrow{OE}$ = $\frac{1}{3}$**a**.

(a) On a single diagram, illustrate the points *A, B, C, D* and *E*. [3]

(b) Given that *M* is the midpoint of *OD*,

(i) express $\overrightarrow{EC}$ and $\overrightarrow{EM}$ in terms of **a** and **b**; [2]

(ii) deduce that *E, M* and *C* are collinear. [1]

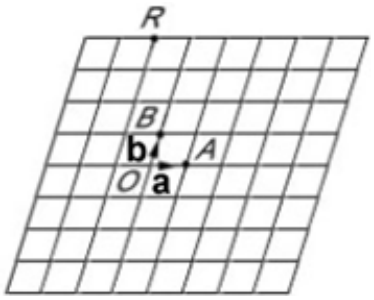
16. It is given that $\overrightarrow{PQ} = \begin{pmatrix} 12 \\ -9 \end{pmatrix}$.

(a) Calculate $|\overrightarrow{PQ}|$. [1]

(b) Given that *PQRS* is a parallelogram, express $\overrightarrow{RS}$ as a column vector. [1]

(c) Given that *P* is the point (–2, 5) and that *M* is the midpoint of *PQ*, find the coordinates of *M*. [2]

17. On the grid given, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.
The point *R* is also shown.



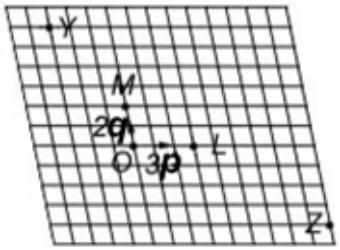
(a) Mark clearly on the grid

(i) the point *P*, such that $\overrightarrow{OP} = 3\mathbf{a} + 2\mathbf{b}$; [1]

(ii) the point *Q*, such that $\overrightarrow{OQ} = -3(\mathbf{a} - \mathbf{b})$. [1]

(b) Write down $\overrightarrow{OR}$ in terms of **a** and **b**. [1]

18. On the grid given, $\overrightarrow{OL} = 3\mathbf{p}$ and $\overrightarrow{OM} = 2\mathbf{q}$.
The points *Y* and *Z* are also shown.



(a) Mark clearly on the grid

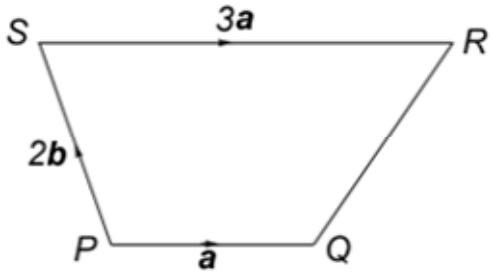
(i) the point *U*, such that $\overrightarrow{OU} = -(3\mathbf{p} + 4\mathbf{q})$, [1]

(ii) the point *V*, such that $\overrightarrow{OV} = 3(3\mathbf{p} + 2\mathbf{q})$. [1]

(b) Write down $\overrightarrow{OY}$ and $\overrightarrow{OZ}$ in terms of **p** and **q**.[2]

(c) Find $\overrightarrow{YZ}$ in terms of **p** and **q**. [1]

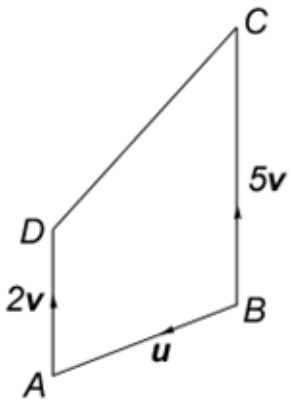
19. Write each of the following vectors in terms of **a** and/or **b**:



(a) $\overrightarrow{QP}$, [1] (b) $\overrightarrow{QS}$, [1]

(c) $\overrightarrow{QR}$, [1] (d) $\overrightarrow{PR}$. [1]

20. Express each of the following vectors in terms of **u** and/or **v**:



(a) $\overrightarrow{CB}$, [1] (b) $\overrightarrow{AC}$, [1]

(c) $\overrightarrow{DC}$, [1] (d) $\overrightarrow{BD}$. [1]

21. It is given that $\mathbf{a} = \begin{pmatrix} -5 \\ -12 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} -8 \\ 6 \end{pmatrix}$, $\mathbf{c} = \begin{pmatrix} t \\ 9 \end{pmatrix}$.

(a) Express $2\mathbf{b} - \mathbf{a}$ as a column vector. [1]

(b) Find $|\mathbf{a}|$ and $|\mathbf{b}|$. [2]

(c) Given that **c** is parallel to **b**, find the value of *t*. [1]

22. *O* is the origin, *A* is the point (–1, –3) and *B* is the point (–3, 7).

(a) Given the vector $\overrightarrow{OC}$ is such that $\overrightarrow{OC} = 2\overrightarrow{OA} + 3\overrightarrow{OB}$, find

(i) the coordinates of *C*; [1]

(ii) the value of $\left|\overrightarrow{OC}\right|$, correct to three significant figures. [1]

(b) Given that *D* is the point (1, *a*) and that $\overrightarrow{OD} = b\overrightarrow{OA} - 2\overrightarrow{OB}$, find the values of *a* and *b*. [2]

23. The column vectors **p, q, r** and **s** are defined by

$\mathbf{p} = \begin{pmatrix} 9 \\ 3 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} -2 \\ 6 \end{pmatrix}$, $\mathbf{r} = \begin{pmatrix} 1 \\ -5 \end{pmatrix}$ and $\mathbf{s} = \begin{pmatrix} a \\ b \end{pmatrix}$.

(a) Express each of the following as a column vector:

(i) $\frac{1}{3}\mathbf{p} - 2\mathbf{q}$, [1]

(ii) $\mathbf{p} + \frac{1}{2}\mathbf{q} - 3\mathbf{r}$. [1]

(b) Evaluate each of the following, giving the answer correct to one decimal place:

(i) $|\mathbf{p} - \mathbf{q}|$, [2]

(ii) $|\mathbf{q} + 4\mathbf{r}|$. [2]

(c) Given that $\mathbf{p} - \mathbf{s} + 2\mathbf{r} = \mathbf{s} - 3\mathbf{q}$, find the values of *a* and *b*. [2]

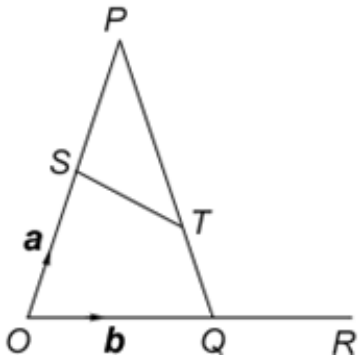
24. $\overrightarrow{AB} = \begin{pmatrix} -5 \\ 3 \end{pmatrix}$, $\overrightarrow{BC} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$, $\overrightarrow{CD} = \begin{pmatrix} -5 \\ 12 \end{pmatrix}$. Find

(a), $\overrightarrow{AC}$ [1] (b) $\overrightarrow{AD}$, [1]

(c) $\overrightarrow{DE}$, given that $4\overrightarrow{DE} = \overrightarrow{CB}$, [1] (d) $|\overrightarrow{CD}|$. [1]

25. In the diagram, $\overrightarrow{OS}$ = **a** and $\overrightarrow{OQ}$ = **b**.

$OS = \frac{1}{2}OP$, $OR = 2OQ$ and $PT = \frac{2}{3}PQ$.



(a) Express each of the following vectors as simply as possible in terms of **a** and/or **b**:

(i) $\overrightarrow{PQ}$, [1]

(ii) $\overrightarrow{TQ}$, [1]

(iii) $\overrightarrow{OT}$, [1]

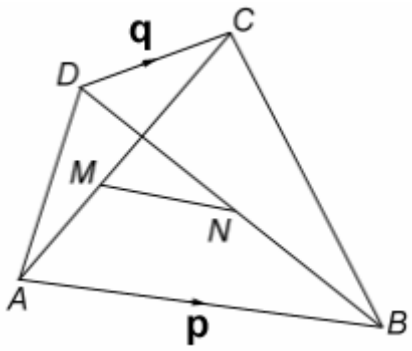
(iv) $\overrightarrow{ST}$, [1]

(v) $\overrightarrow{SR}$. [1]

(b) Prove that *ST*, when produced, will pass through *R*. [2]

(c) Find the ratio *SR : TR*. [1]

26. In the diagram, *M* and *N* are the midpoints of the diagonals *AC* and *BD* respectively. Given that $\overrightarrow{AB}$ = **p** and that $\overrightarrow{DC}$ = **q**, find the vector *MN* in terms of **p** and **q**.

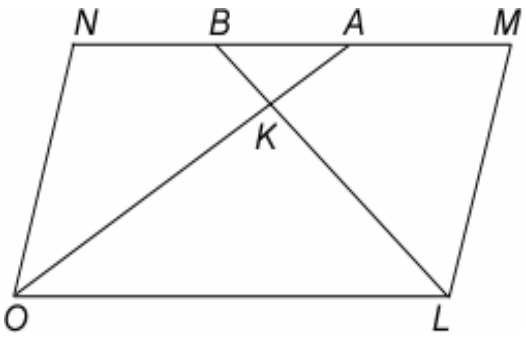


27. Given that $\overrightarrow{PQ} = \frac{1}{7}\overrightarrow{PR}$ and $\overrightarrow{SR} = \frac{1}{2}\overrightarrow{QR}$, express $\overrightarrow{PS}$ in terms of $\overrightarrow{PR}$.

28. In the diagram, *OLMN* is a parallelogram and

$\overrightarrow{AB} = \overrightarrow{MA} = \overrightarrow{BN}$.

Express $\overrightarrow{OK}$ in terms of $\overrightarrow{OL}$ and $\overrightarrow{OB}$.



29. Given that **a** = $\begin{pmatrix} \sqrt{3} \\ 1 \end{pmatrix}$ and that **b** = $\begin{pmatrix} 3\sqrt{3} \\ -5 \end{pmatrix}$, find

(a) the value of |**a** + **b**|;
(b) the direction of the vector **a** + **b**.

30. A triangle *ABC* has vertices at *A* (6, 2), *B* (4, 4) and *C* (–2, –4).

(a) Express $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{CA}$ as column vectors.

(b) Find $|\overrightarrow{AB}|$, $|\overrightarrow{BC}|$ and $|\overrightarrow{CA}|$.
(c) Write down an important fact about triangle *ABC*.

31. *PQRS* is a quadrilateral in which *P, Q, R* and *S* are the points (–4, –1), (–3, –2), (0, 1) and (–1, 2) respectively.

(a) Consider the pair of vectors $\overrightarrow{PQ}$ and $\overrightarrow{SR}$. Write down two important facts about the pair of sides *PQ* and *SR*.
(b) Consider the pair of vectors $\overrightarrow{PS}$ and $\overrightarrow{QR}$. Write down two important facts about the pair of sides *PS* and *QR*.
(c) Consider the magnitudes of the pair of vectors $\overrightarrow{PR}$ and $\overrightarrow{QS}$. Write down an important fact about the pair of sides *PR* and *QS*.
(d) Hence, write down an important fact about the quadrilateral *PQRS*.

32. *O* is the origin, *A* is the point (2, –3), *B* is the point (6, 5), *C* is the point (–2, 1) and *D* is the point (–6, –7).

(a) Consider the pair of vectors $\overrightarrow{AB}$ and $\overrightarrow{DC}$. Write down two important facts about the pair of sides *AB* and *DC*.

(b) Consider the pair of vectors $\overrightarrow{AD}$ and $\overrightarrow{BC}$. Write down two important facts about the pair of sides *AD* and *BC*.

(c) Write down an important fact about the sides *AB, BC, DC* and *AD*.

(d) Show that $\left|\overrightarrow{AC}\right| \neq \left|\overrightarrow{BD}\right|$.

(e) Hence, write down an important fact about quadrilateral *ABCD*.

33. Given that $\mathbf{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ and that $\mathbf{b} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$, find the values of $|a + b|$, $|3a + 2b|$ and $|4a - 5b|$.

34. Solve the equations:

(a) $3\mathbf{x} + \begin{pmatrix} 4 \\ -1 \end{pmatrix} = \begin{pmatrix} 5 \\ 9 \end{pmatrix} - 2\mathbf{x}$,

(b) $4\mathbf{x} + 2\begin{pmatrix} 3 \\ 1 \end{pmatrix} + \begin{pmatrix} -2 \\ 4 \end{pmatrix} = 0$.

Answers

1. (a) 10 units (b) (i) $\begin{pmatrix} 4 \\ -7 \end{pmatrix}$ (ii) $\begin{pmatrix} 0 \\ 8 \end{pmatrix}$ (c) $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$

2. $\begin{pmatrix} -6 \\ -1 \end{pmatrix}$

3. (a) $-\mathbf{p}$ (b) $-\frac{1}{2}\mathbf{r}$ (c) $\mathbf{r}-\mathbf{p}$ (d) $\mathbf{p}+\frac{1}{2}\mathbf{r}$

4. (a) $\begin{pmatrix} 12 \\ 8 \end{pmatrix}$ (b) $\begin{pmatrix} -3 \\ -2 \end{pmatrix}$ (c) $\begin{pmatrix} 9 \\ 6 \end{pmatrix}$

5. (a) $3\mathbf{s}$ (b) (i) $\frac{9}{7}$ (ii) $\frac{3}{4}$

6. (a) $\begin{pmatrix} -7 \\ -15 \end{pmatrix}$ (b) (i) 7.5 (ii) $\begin{pmatrix} -21.5 \\ -7.5 \end{pmatrix}$ (c) 9 units

7. (a) $\mathbf{p}-\mathbf{q}$ (b) $\frac{1}{3}\mathbf{p}+\frac{2}{3}\mathbf{q}$

8. (a) 13 units (b) (16, –12) (c) $\begin{pmatrix} -24 \\ 10 \end{pmatrix}$ or $\begin{pmatrix} 24 \\ -10 \end{pmatrix}$

9. (a) (i) $2\mathbf{a}+3\mathbf{b}$ (ii) $3\mathbf{a}+3\mathbf{b}$ (iii) $\mathbf{b}-\mathbf{a}$ (iv) $3\mathbf{a}+2\mathbf{b}$
   (b) $(3\mathbf{k}-1)\,\mathbf{a}+(2\mathbf{k}+1)\mathbf{b}$

10. (a) (i) $\begin{pmatrix} -10 \\ 15 \end{pmatrix}$ (ii) $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$ (b) (–1, 7)

11. (a) (i) $\mathbf{l}=1$, $\mathbf{m}=-4$ (b) (i) –5 (ii) $\begin{pmatrix} 10 \\ -13 \end{pmatrix}$

12. (a) $\mathbf{k}=3$, $\mathbf{h}=-4$ (b) $\frac{1}{2}\mathbf{b}-\mathbf{a}$

13. (a) (i) $2\mathbf{a}+\mathbf{b}$ (ii) $2\mathbf{b}+\mathbf{a}$ (iii) $\mathbf{b}-\mathbf{a}$ (iv) $\mathbf{b}-\mathbf{a}$

14. (a) (1, –3) (b) $\mathbf{m}=2$, $\mathbf{n}=3$

15. (b) (i) $\overrightarrow{EC} = \frac{2}{3}\mathbf{a} + \mathbf{b}$, $\overrightarrow{EM} = \frac{1}{6}\mathbf{a} + \frac{1}{4}\mathbf{b}$

16. (a) 15 units (b) $\begin{pmatrix} -12 \\ 9 \end{pmatrix}$ (c) $(4, \frac{1}{2})$

17. (b) $4\mathbf{b} - \mathbf{a}$

18. (b) $\overrightarrow{OY} = 6\mathbf{q} - 3\mathbf{p}$, $\overrightarrow{OZ} = 9\mathbf{p} - 4\mathbf{q}$ (c) $2(6\mathbf{p} - 5\mathbf{q})$

19. (a) $-\mathbf{a}$ (b) $2\mathrm{b} - \mathbf{a}$ (c) $2(\mathbf{a} + \mathbf{b})$ (d) $3\mathbf{a} + 2\mathbf{b}$

20. (a) $-5\mathbf{v}$ (b) $5\mathbf{v} - \mathbf{u}$ (c) $\mathbf{3v} - \mathbf{u}$ (d) $\mathbf{u} + 2\mathbf{v}$

21. (a) $\begin{pmatrix} -11 \\ 24 \end{pmatrix}$ (b) $|\mathbf{a}| = 13$ units, $|\mathbf{b}| = 10$ units (c) $\mathbf{t} = -12$

22. (a) (i) $(-11, 15)$ (ii) 18.6 units (b) $\mathbf{a} = -29$, $\mathbf{b} = 5$

23. (a) (i) $\begin{pmatrix} 7 \\ -11 \end{pmatrix}$ (ii) $\begin{pmatrix} 5 \\ 21 \end{pmatrix}$ (b) (i) 11.4 units (ii) 14.1 units (c) $\mathbf{a} = 2\frac{1}{2}$, $\mathbf{b} = 5\frac{1}{2}$

24. (a) $\begin{pmatrix} -3 \\ -1 \end{pmatrix}$ (b) $\begin{pmatrix} -8 \\ 11 \end{pmatrix}$ (c) $\begin{pmatrix} -\frac{1}{2} \\ 1 \end{pmatrix}$ (d) 13 units

25. (a) (i) $\mathbf{b} - 2\mathbf{a}$ (ii) $\frac{1}{3}(\mathbf{b} - 2\mathbf{a})$ (iii) $\frac{2}{3}(\mathbf{a} + \mathbf{b})$ (iv) $\frac{1}{3}(2\mathbf{b} - \mathbf{a})$
(v) $2\mathbf{b} - \mathbf{a}$
(c) 3 : 2

26. $\frac{1}{2}(\mathbf{p} - \mathbf{q})$

27. $\frac{4}{7}\overrightarrow{PR}$

28. $\frac{1}{4}(\overrightarrow{OL} + 3\overrightarrow{OB})$

29. (a) 8 units (b) 330°

30. (a) $\begin{pmatrix} -2 \\ 2 \end{pmatrix}, \begin{pmatrix} -6 \\ -8 \end{pmatrix}, \begin{pmatrix} 8 \\ 6 \end{pmatrix}$ (b) 2.8 units, 10 units, 10 units (c) Triangle *ABC* is isosceles.

31. (a) *PQ* and *SR* are parallel and have equal lengths.
(b) *PS* and *QR* are parallel and have equal lengths.
(c) *PR* and *QS* have equal lengths.
(d) *PQRS* is a rectangle.

32. (a) *AB* and *DC* are parallel and have equal lengths.
(b) *AD* and *BC* are parallel and have equal lengths.
(c) *AB*, *BC*, *DC* and *AD* have equal lengths.
(d) *ABCD* is a rhombus.

33. 1.4 units, 6.4 units, 25.6 units.

34. (a) $\begin{pmatrix} 0.2 \\ 2 \end{pmatrix}$ (b) $\begin{pmatrix} -1 \\ -1.5 \end{pmatrix}$

Chapter 4

Secondary 4 Mathematics
Chapter 4 Standard Deviation and Mean

GENERAL NOTES

At the beginning of this chapter, teachers should revise with their students the measures of central tendency so that they can gain a better idea of these measures and appreciate better the usefulness of these measures in comparing two sets of data. For example, the performance of pupils in class A in a test may be better than pupils in class B. Ask the students how we can find out about this. Ask them whether it is because every pupil in class A scored higher marks than every pupil in class B. In the discussion, you may point out to them that there are bound to be some pupils in class A who scored very low marks as well as some pupils in class B who scored relatively very high marks. But these exceptions will be very few and it will be found that the marks of most pupils in each class will approximate to the average value for the whole of that class. Apart from those pupils with very low and high marks, the marks of other pupils in each class tend to cluster around a central value, and it is this clustering which gives the average its meaning. Thus, the average as a central value may be taken as being representative of all the marks of students of the whole class. By comparing the representative figures of the two sets of marks, we may be able to conclude that the performance of pupils in one class in a test is better than the other class.

Point out to students that computing the mean for a grouped frequency distribution is similar to computing the mean of an ungrouped frequency distribution. But since the compression of data in a grouped frequency table results in the loss of the actual values of the observations in each class in the frequency column, it is necessary to make an assumption about these values. The assumption is that every observation in a class has a value equal to the mid-value of the class. Thus, the mean of a grouped frequency distribution is an estimate of the mean of the original observations.

We use standard deviation to measure the dispersion of a set of data from its mean. Generally, the more widespread a set of data is, the higher the deviation. Standard deviation is useful when comparing the spread of two data sets that have approximately the same mean.

To add interest to the study of statistics, teachers can give mini-projects for groups of pupils asking them to collect data concerning students in the class, concerning the school, concerning anything of interest to the students and so on. Using these data, students can practise what they have learnt in organising data into grouped frequency distribution table, presenting the distribution in the form of a histogram and a frequency polygon and calculating the mean and standard deviation of the distribution. Hence compare and comment on the dispersion of the data.

XYZ SECONDARY SCHOOL

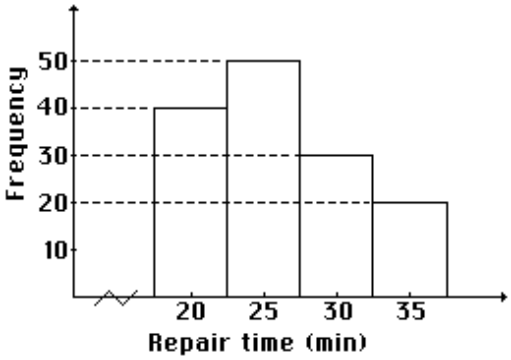
Name: _____ ( ) Date: _____

Time allowed: min

Class: _____ Marks: /10

Secondary 4 Multiple-Choice Questions
Chapter 4 Standard Deviation and Mean

1. The diagram shows a histogram representing the length of time taken to repair a machine in a factory. Which of the following is true?



(I) The class boundaries of the modal class are 22.5 min and 27.5 min.
(II) The mean repair time is 25.5 min correct to the nearest 0.1 min.
(III) The median repair time is very near to 25 min.

(A) I only (B) I and II (C) I and III
(D) II and III (E) I, II and III ( )

2. The following table gives the collar-size of the shirts of 200 men.

Size (cm)	32	34	36	38	40	42
Frequency	34	46	62	26	18	14

Which of the following about the data is most important to a manufacturer of shirts?

(A) The mean (B) The median (C) The mode
(D) The smallest collar-size (E) The biggest collar size ( )

3. What is the mean collar-size in Qn. 2?

(A) 35.5 cm (B) 35.9 cm (C) 36.3 cm
(D) 36.7 cm (E) 37.1 cm ( )

4. Consider the following set of data : 3, 4, 5, 5.
If each number in the set is doubled, which of the following quantity or quantities will be doubled?
I. Mean II. Median III. Mode

(A) I only (B) II only (C) III only
(D) II and III (E) I, II and III ( )

5. The mean of 10 numbers is 25. Given that one of the numbers is 70, find the mean of the other 9 numbers.

(A) 95 (B) 45 (C) 20 (D) 7 (E) 5 ( )

6. The mean of a and b is x, the mean of b and c is y and the mean of a and c is z. Express the mean of a, b and c in terms of x, y and z.

(A) $\frac{1}{6}(x + y + z)$ (B) $\frac{1}{3}(x + y + z)$ (C) $\frac{1}{2}(x + y + z)$
(D) $\frac{2}{3}(x + y + z)$ (E) $1\frac{1}{2}(x + y + z)$ ( )

7. Considering the set of data : 5, 3, 6, 4, 5, 5, 8, 8, 7, 7, which of the following is/are true?

I. The mean is the biggest.
II. II. The median is the smallest.
III. The average of the median and mean is smaller than the mode.
IV. The average of the median and mode is smaller than the mean.

(A) I only (B) I and III (C) I, III and IV
(D) II and IV (E) I and IV ( )

8. The frequency distribution of the height of 100 students is given in the following table. Find the mean height of the students.

Height (x cm)	$110 < x \leq 114$	$114 < x \leq 118$	$118 < x \leq 122$	$122 < x \leq 126$	$126 < x \leq 130$
No. of students	7	10	44	29	10

(A) 119.5 (B) 120 (C) 120.5
(D) 121 (E) 121.5 ( )

9. Calculate the standard deviation of the set of marks below.
9 , 10 , 2 , 2 , 5 , 6 , 3 , 7 , 10 , 6

(A) 2.2 (B) 1.89 (C) 2.86
(D) 2.9 (E) 1.73 ( )

10. Find the standard deviation of the scores below.
17 , 14 , 1 , 1 , 3 , 0 , 2 , 26

(A) 8.87 (B) 9.11 (C) 9.23
(D) 8.91 (E) 7.98 ( )

Answers

1. C	**2.** C	**3.** B	**4.** E	**5.** C
6. B	**7.** E	**8.** D	**9.** D	**10.** B

XYZ SECONDARY SCHOOL

Name: _____ ( ) Date: _____

Time allowed: min

Class: _____ Marks: 

Secondary 4 Mathematics Test
Chapter 4 Standard Deviation and Mean

1. The following are the scores of thirteen students in a mathematics test.
1, 5, 3, 12, 5, 19, 16, 8, 17, 20, 8, 5, 11

(a) State the mode. [1]
(b) Find the median score. [2]
(c) Calculate the mean score of the test. [2]
(d) When the number x is added to the above set, the new mean is 10.5. Calculate the value of x. [2]

2. The number of eggs laid each day by 18 hens for the period of 21 days were:
16, 17, 18, 16, 15, 18, 16, 18, 16, 15, 16, 14, 17, 16, 18, 17, 15, 17, 16, 17, 15

(a) State the modal number of eggs laid per day. [1]
(b) Find the median number of eggs laid per day. [2]
(c) Calculate the mean number of eggs laid per day. [2]

3. A driver keeps records of his average mileage per litre, recording his findings to the nearest integer. His first 25 results are given below.

14 17 16 15 16 17 19 17 15 16 16 17 16
14 15 16 18 16 15 16 17 16 18 16 17

Copy and complete the following table.

km/litre	14	15	16	17	18	19
Frequency	2					1

[2]

(a) Write down the mode. [1]
(b) Find the median average mileage per litre. [1]
(c) Calculate the mean average mileage per litre. [2]

4. A band master keeps a record of the numbers turning up for band practice for eight chosen sessions. The mean attendance of the eight sessions is 27.5.
Given that the attendance for six of the sessions are
22, 24, 25, 29, 24 and 25,
find the mean attendance of the other two sessions. [3]

5. The systolic blood pressure (in standard units) of 15 smokers are as follows:
129, 145, 124, 117, 148, 130, 120, 132, 115, 136, 120, 130, 125, 144, 120

(a) Write down the mode. [1]
(b) Find the median pressure. [2]
(c) Calculate the mean pressure of the 15 smokers. [3]

6. The systolic blood pressures of 30 non-smokers are given in the table below.

Systolic blood pressure	110	111	112	113	114	115	116	117
Frequency	1	3	2	5	5	8	4	2

(a) Find the median pressure. [2]
(b) Calculate the mean pressure. [3]
(c) Calculate the standard deviation. [3]

7. The distance, x km, travelled by a group of people to work each day were given by the table below.

Systolic blood pressure	3	4	6	8	14
Frequency	15	8	4	2	1

(a) Find the mean distance travelled. [3]
(b) If the different distances travelled are represented on a pie chart, calculate the number of degrees in the sector representing $x = 4$. [2]
(c) Calculate the standard deviation. [3]

8.

Score	5	6	7	8
Frequency	4	7	x	6

For the above frequency distribution, find the largest and smallest possible values of x such that the median is 6. [3]

9. A bag contains nails of the following sizes:

Length (in mm)	10	15	20	25	30	35	40
No. of nails	12	14	24	17	12	13	8

(a) State the modal length of the nails. [1]
(b) Calculate the mean length of the nails. [3]
(c) Find the proportion of nails whose length is longer than the mean length. [2]
(d) Calculate the standard deviation. [3]

10. The distribution of the weights of 30 Primary Four boys is shown in the table below.

Weight (kg)	32	33	34	35	36
No. of boys	4	5	7	9	5

Calculate the
(a) mode, **(b)** median and **(c)** mean weight of the boys. [3]

11. The following table shows the amount of weight lost by 100 women after a slimming course of 4 weeks.

Loss in kg	0	1	2	3	4	5	6	7	8
Frequency	3	6	11	19	23	25	8	3	2

Find the mode, median, mean and standard deviation. [6]

12. The following scores were recorded in a test :
3, 7, 8, 6, 4, 7, 6, 8, 3, 5, 8, 9, 8, 5, 10.

Calculate the following.
(a) the mode [1]
(b) the median [2]
(c) the mean [2]
(d) the percentage of pupils who scored more than 5 marks. [2]

13. The daily wages of 60 factory workers are shown in the table below.

Daily wage ($ x)	$20 \le x < 22$	$22 \le x < 24$	$24 \le x < 26$	$26 \le x < 28$	$28 \le x < 30$	$30 \le x < 32$
Frequency	10	15	12	10	8	5

(a) State the modal class. [1]
(b) Calculate an estimate for the mean daily wage. [4]

14. The heights of 100 trees were measured with the following results.

Height (h metres)	$0.4 < h \le 0.8$	$0.8 < h \le 1.2$	$1.2 < h \le 1.6$	$1.6 < h \le 2.0$	$2.0 < h \le 2.4$
Frequency	10	22	45	18	5

(a) State the modal class. [1]
(b) Find the range of heights in which the median lies. [2]
(c) Calculate the standard deviation. [3]

15. The distribution of the cost, $$x$, of fifty pairs is shown in the following table.

Cost ($ x)	$30 \le x < 40$	$40 \le x < 50$	$50 \le x < 60$	$60 \le x < 70$	$70 \le x < 80$	$80 \le x < 100$
Frequency	5	6	12	15	4	8

(a) Draw a histogram to represent the data. [4]
(b) Calculate the standard deviation. [3]

16. The lengths of stay of cars at a car park were recorded and the results are shown in the table below.

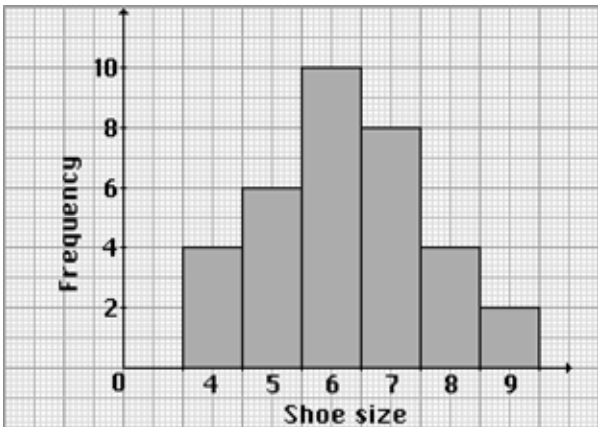
Time (t minutes)	$0 < t \le 15$	$15 < t \le 30$	$30 < t \le 40$	$40 < t \le 50$	$50 < t \le 80$	$80 < t \le 120$
Frequency	48	66	86	72	36	32

(a) Draw a histogram to represent the data. [4]
(b) Calculate an estimate for the mean length of stay at the car park. [4]
(c) Calculate the proportion of cars that stay at the car park for at most 40 minutes. [2]

17. (a) The median of a set of eight numbers is 6. Given that seven of the numbers are 8, 9, 2, 3, 14, 5, 1, find the eighth number. [2]

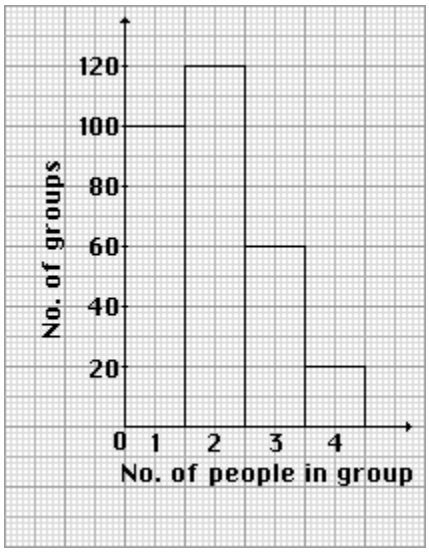
(b) The mean of a set of seven numbers is 4, and the mean of another set of thirteen numbers is x.
Given that the mean of the combined set of twenty numbers is 6.6, calculate x. [3]

18. The bar-chart shows the size of shoes worn by x pupils in a class.



(a) Find x. [1]
(b) State the modal shoe size. [1]
(c) The price of a pair of shoes is directly proportional to its size. Calculate the average price of a pair of shoes worn by x pupils, if a size 6 pair costs \$12 while a size 8 pair costs \$16. Give your answer correct to 1 decimal place. [3]

19. The diagram shows the number of people in groups of 1, 2, 3 and 4 who patronised a hawker centre during a five-hour period. Calculate



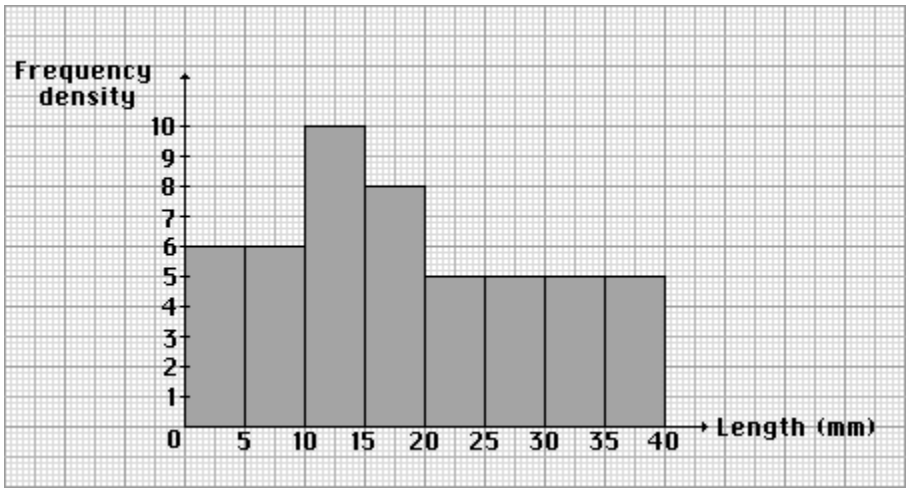
(a) the total number of people who patronised the hawker centre, [1]
(b) the angle, in a pie chart, of the sector which represents groups with 3 people, [1]
(c) the percentage of groups with at least 2 people, [1]
(d) the mean number of people patronising the hawker centre per hour. [1]

20. In a test, each pupil of a group scores 5, 10 or 15 marks. The number of pupils scoring each mark is shown in the table below.

Marks	5	10	15
No. of pupils	8	12	x

(a) If the mode is 10, write down the range of values of x. [1]
(b) If the median mark is 10, write down the largest possible value of x. [1]
(c) Using the value of x found in (ii), calculate the mean mark. [2]

21. The histogram illustrates lengths of nails in a bag.



(a) Copy and complete the following table:

Length (in mm)	$0 < x \leq 10$	$10 < x \leq 15$	$15 < x \leq 20$	$20 < x \leq 40$
No. of nails		10	8	

[2]

(b) Find, in its simplest form, the fraction of nails in the bag that have longer lengths than 15 mm. [1]
(c) Calculate an estimate for the mean length of a nail. [3]

22. The marks of ten students in a test are 9, 14, 18, 23, 24, 26, 30, 31, 34, 38.
(a) Find the mean mark. [2]
(b) The marks of another ten students sitting for the same test are given below.
1, 6, 10, 15, 16, 18, 22, 23, 26, 30.
(i) Write down the mean mark for these ten students. [1]
(ii) Calculate the mean mark for the twenty students. [2]

23. 100 sentences were taken from a book and the number of words per sentence were counted.

(a) Copy and complete the following table.

No. of words per sentence	Mid-value (x)	No. of sentences (f)	fx	fx^2
1 – 5	3	16		
6 – 10	8	22		
11 – 15	13	18		
16 – 20		11		
21 – 25		12		
26 – 30		9		
30 – 35		8		
36 – 40		2		
41 – 45		2		
		$\sum f = 100$	$\sum fx =$	$\sum fx^2 =$

[4]

(b) Hence, calculate the standard deviation. [3]

24. The daily wages of 100 construction workers are displayed in the table below.

Daily wage (\$ x)	$16 \le x < 18$	$18 \le x < 20$	$20 \le x < 22$	$22 \le x < 24$	$24 \le x < 26$	$26 \le x < 28$	$28 \le x < 30$
Frequency	8	10	18	30	22	7	5

(a) Write down the modal class. [1]

(b) Calculate the mean daily wage

(i) without using an assumed mean, [3]

(ii) using an assumed mean. [3]

25. A school librarian recorded the number of books borrowed weekly by pupils in one particular class during 40 successive weeks. The results are shown in the table below.

No. of books borrowed	10 – 14	15 – 19	20 – 24	25 – 29	30 – 34	35 – 39
No. of weeks	5	7	9	8	5	6

(a) Write down the modal class. [1]

(b) Draw a histogram to illustrate this information. [4]

(c) Calculate the mean number of books borrowed per week. [3]

26. In an experimental farm, 30 hectare plots of land growing potatoes produced yields in tonnes as shown in the grouped frequency table below.

Yield (x tonnes)	$3.4 \le x < 3.6$	$3.6 \le x < 4.0$	$4.0 \le x < 4.4$	$4.4 \le x < 4.8$	$4.8 \le x < 5.0$	$5.0 \le x < 5.6$
No. of plots	3	6	8	5	6	2

(a) Draw a histogram to represent the results. [3]
(b) Calculate the mean yield per plot. [3]

27. Consider the following set of eight numbers 8, 10, 14, 5, 7, $12 - x$, $12 + x$, y, where $12 - x$ is a positive integer.

(a) Given that the mean of these eight numbers is 9.5, find the value of y. [2]
(b) Given also that the median of this set is 9, calculate the possible value of x. [2]

28. The table below shows the speeds, arranged in groups, of 100 cars passing a check point.

Speed (x km/h)	$40 < x \le 50$	$50 < x \le 52$	$52 < x \le 56$	$56 < x \le 60$	$60 < x \le 80$
No. of cars	20	30	a	15	5

If the mean speed is 52.7 km/h, calculate the value of a. [4]

29. The table below shows the distribution of marks obtained in a Mathematics test taken by all the Primary 4 students in a school.

Marks	0	1	2	3	4	5	6	7	8	9	10
Number of students	3	7	14	26	32	54	67	53	25	16	12

Calculate
(a) the total number of students who sat for the test, [2]
(b) the mode, median and mean score. [3]

30. The heights of 100 children were measured and the results shown in the table below.

Height (cm)	64	65	66	67	68	69	70	71	72	73	74
Number of children	1	2	3	10	16	15	19	20	10	2	2

Calculate
(a) the mode and the median, [4]
(b) the mean height. [2]

31. The distribution of the price, correct to the nearest dollar, of each of 40 pairs of shoes is shown in the following table.

Price ($)	50 – 54	55 – 59	60 – 64	65 – 69	70 – 74
Frequency	5	10	12	5	8

(a) Draw a histogram to represent these data. [3]
(b) Calculate the standard deviation. [3]

32. (a) The ages of 30 men convicted for the first time of violent crime in Country *X* gave the following figures.

22	32	29	28	22	16	19	17	17	16
19	18	18	30	20	20	28	28	20	23
23	35	19	22	21	17	32	23	30	21

(i) Arrange the data in classes of 15 – 19, 20 – 24, …, 34 – 39. [3]
(ii) Estimate the mean age. [2]

(b) The grouped frequency distribution of the ages of a group of men convicted for the first time of violent crime from Country *Y* is as shown in the table below.

Age (x years)	$16 \le x < 18$	$18 \le x < 20$	$20 \le x \le 25$	$25 \le x < 28$	$28 \le x \ 30$	$30 \le x < 40$
Frequency	12	10	23	15	8	12

(i) Draw a histogram to represent this information. [3]
(ii) Estimate the mean age. [2]
(iii) Comparing the answers in (a)(ii) and (b)(ii), draw a conclusion concerning the ages of men convicted for the first time of violent crime in Country *X* and Country *Y*. [2]

33. The diagram of 50 ball bearings by a factory measured in mm (correct to 2 significant figures) are given in the table below.

Diameter (mm)	5.0 – 5.2	5.3 – 5.5	5.6 – 5.8	5.9 – 6.1	6.2 – 6.4	6.5 – 6.7
Frequency	6	8	12	14	7	3

(i) State the modal class. [2]
(ii) Find the mean. [2]
(iii) Calculate the standard deviation. [2]

34. The diameters of 50 test tubes produced by a glass factory measured in mm (correct to 2 significant figures) are given in the table below.

Diameter (mm)	5.8 – 6.0	6.1 – 6.3	6.4 – 6.6	6.7 – 6.9	7.0 – 7.2	7.3 – 7.5
Frequency	6	8	12	14	7	3

(i) State the modal class. [2]
(ii) Find the mean. [2]
(iii) Calculate the standard deviation. [2]

35. The students in two classes took the same test. Information relating to the results is shown in the tables below.

Class A

Marks	1-5	6-10	11-15	16-20	21-25
Frequency	3	6	10	7	4

Class B

Mean = 3.5
Standard Deviation = 6.9

(a) For Class A, calculate **(i)** the mean [2]
(ii) the standard deviation. [2]

(b) Compare, briefly, the results for the two classes. [2]

Answers

1. (a) 5 (b) 8 (c) 10 (d) 17

2. (a) 16 (b) 16 (c) $16\frac{1}{3}$

3. 2, 4, 10, 6, 2, 1 (a) 16 km/litre (b) 16 km/litre (c) 16.2 km/litre

4. 35.5

5. (a) 120 (b) 129 (c) 129

6. (a) 114 (b) 114 (c) 1.81

7. (a) 4.4 km (b) 96° (c) 2.24

8. 4, 0

9. (a) 20 (b) 23.7 (c) $\frac{1}{2}$ (d) 8.91

10. (a) 35 (b) 34 (c) 34.2

11. 5, 4, 3.9, 1.62

12. (a) 8 (b) 7 (c) 6.5 (d) $66\frac{2}{3}$ %

13. (a) $22 \leq x < 24$ (b) $ 25.20

14. (a) $1.2 < h \leq 1.6$ (b) $1.2 < h \leq 1.6$ (c) 0.52

15. (b) 16.2

16. (b) 40.1 min (c) $\frac{10}{17}$

17. (a) 7 (b) 8

18. (a) 34 (b) 6 (c) $ 12.5

19. (a) 600 (b) 108° (c) $66\frac{2}{3}$ % (d) 120

20. (a) $0 \leq x \leq 11$ (b) 19 (c) 11.4

21. (a) 12, 20 (b) $\frac{14}{25}$ (c) 18.5 cm

22. (a) 24.7 (b) (i) 16.7 (ii) 20.7

23. (b) 10.5

24. (a) $22 \leq x < 24$ (b) 22.78

25. (a) 20 – 24 (c) 24.4

26. (b) 4.33

27. (a) 8 (b) 5, 6, 7, 8, 9, 10, 11

28. 30

29. (a) 309 (b) 6 marks, 6 marks, 5.7 marks

30. (a) 71 cm, 70 cm (b) 69.5 cm

31. (b) 6.47

32. (a) (ii) 22.8 years (b) (ii) 24.5 years
(iii) men convicted for the first time of violent crime from Country *X* were younger on average.

33. (i) 5.9 – 6.1 mm (ii) 5.8 mm (iii) 0.41

34. (i) 6.7 – 6.9 mm (ii) 6.6 mm (iii) 0.41

35. (a) (i) 13.5 (ii) 5.82
(b) B has greater spread of marks

Chapter 5

Secondary 4 Mathematics
Chapter 5 Cumulative Frequency Distribution

ANSWERS FOR ENRICHMENT ACTIVITIES

Just For Fun (pg 189)
Both A and B are murderers.

Just For Fun (pg 206)
In statement (a), suppose 'Jane was first' is true, then in statement (c), either 'Susan was first' is true or 'Jane was fourth' is true. In either, it contradicts the supposition that 'Jane was first' is true. Thus, we conclude that Susan was second and the following table shows the order of the competitors.

	1st	**2nd**	**3rd**	**4th**
Jane	F (a)			T (c)
Karen	T (d)			F (b)
Susan	F (c)	T (a)	F (d)	
Shirley			T (b)	

$\therefore$ Karen : 1^{st} ; Susan : 2^{nd} ; Shirley : 3^{rd} ; Jane : 4^{th}

Secondary 4 Mathematics
Chapter 5 Cumulative Frequency Distribution

GENERAL NOTES

Cumulative frequency diagrams (cumulative frequency polygons and cumulative frequency curves) are another way of representing data graphically. From the diagrams, median, quartiles and percentiles can be found with ease. It should be pointed out to the students that the values obtained from the diagram are merely estimates. For ungrouped data consisting of say, 40 values, the median is the mean of the two middle values (a = 20^{th} value and b = 21^{st} value), when the values are arranged in order of magnitude, i.e. median = $\frac{a + b}{2}$.

An **average** is a number that summarises a set of data. In many cases, the average provides sufficient information, but it fails to give an indication of whether the data are clustered together or spread across a wide range of values. Interquartile range is one measure which will roughly provide such an indication. There are other such measures which are known as measures of dispersion.

If students are given mini-projects to do (e.g. Number of students living in what type of HDB flats or private apartment; water consumption per household etc.), teachers may want them to use the results obtained to construct cumulative frequency tables and draw cumulative frequency diagrams so that they can estimate medians, quartiles, interquartile ranges and so on, from them. Teachers may also give them marks of a class test to analyse by constructing group frequency and cumulative frequency distributions, drawing the cumulative frequency diagram and estimating from the diagram the median mark, quartiles, interquartile range, the percentage of students who passed the test if the pass marks were x marks, the marks above which y% of the students scored and so on. You may also take this opportunity to discuss with them how a set of marks can be moderated by changing the pass marks so that p% of the students will pass, or the range of marks for a certain grade so that q% of the students will obtain a particular grade and so on using the cumulative diagram. If you have the marks of another class of students sitting for the same test, you may want them to analyse as well and make comparison about the performances of the two classes in the test.

For the marks of any future tests, teachers may work out the median and quartiles marks and inform students of these values so that they know where they stand in the class, whether they are in the top 25% or bottom 25% and so on.

A box-and-whisker plot is a simple diagram that gives an idea of the spread of a set of data. It shows the range, the median and the quartiles. It can only make superficial comparisons between the spread of several sets of data.

XYZ SECONDARY SCHOOL

Name: _____ ( ) Date: _____

Time allowed: min

Class: _____ Marks: ⁄ 8

Secondary 4 Multiple-Choice Questions
Chapter 5 Cumulative Frequency Distribution

1. Which of the following is not correct about a set of numbers?

   (A) The mode is the number that occurs with the greatest frequency.
   (B) The quartiles divide the numbers into four equal parts.
   (C) The second quartile is the median.
   (D) Interquartile range = upper quartile – lower quartile.
   (E) Median = $\frac{1}{2}$ (lower quartile + upper quartile) ( )

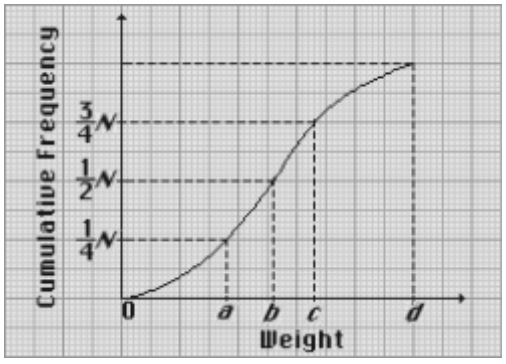
2. Find the interquartile range for the set of numbers 6, 11, 12, 9, 9, 20, 17, 38, 35, 24, 34, 26 and 30.

   (A) 17 (B) 20 (C) 21 (D) 22 (E) 42 ( )

3. The diagram shows a cumulative frequency curve of the weights of N students in a Secondary 3 class. The interquartile range of the distribution is

   (A) $b - a$ (B) $c - b$
   (C) $d - c$ (D) $c - a$
   (E) $d - b$

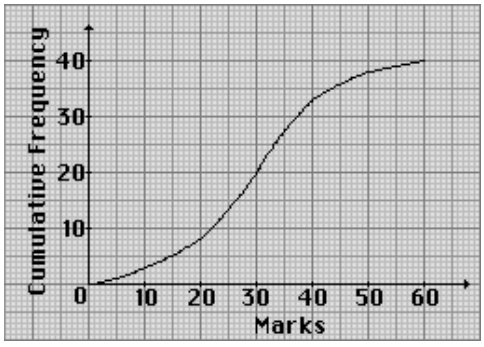
   ( )



4. The diagram shows a cumulative frequency curve for the distribution of marks of a mathematics test for 40 pupils. If 40% of the pupils pass, what is the passing mark of the test?

   (A) 28 (B) 30
   (C) 32 (D) 36
   (E) 44

   ( )



5. Referring to the cumulative frequency curve in Qn. 4, find the interquartile range.

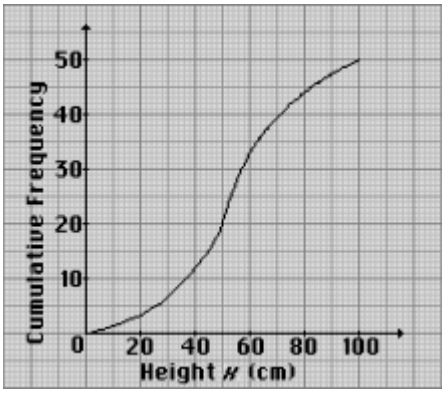
(A) 6 (B) 8 (C) 14 (D) 24 (E) 30 ( )

6. Referring to the cumulative frequency curve in Qn. 4, what is the percentage of pupils who score at least 40 marks?

(A) $7\frac{1}{2}$ % (B) 15 %(C) $42\frac{1}{2}$ % (D) 75 % (E) 85 % ( )

7. The diagram shows the cumulative frequency curve of the heights of 50 plants grown under experimental conditions. Which of the following classes has the highest frequency?

(A) $0 < x \leq 20$ (B) $20 < x \leq 40$
(C) $40 < x \leq 60$ (D) $60 < x \leq 80$
(E) $80 < x \leq 100$



( )

8. Referring to the cumulative frequency curve in Qn. 7, what is the percentage of plants whose heights lie in the range $40 < x \leq 80$?

(A) 30 (B) 32 (C) 40 (D) 60 (E) 64 ( )

Answers

1. E 2. D 3. D 4. C
5. C 6. B 7. C 8. E

XYZ SECONDARY SCHOOL

Name: _____ ( ) Date: _____

Time allowed: min

Class: _____ Marks:

Secondary 4 Mathematics Test

Chapter 5 Cumulative Frequency Distribution

1.

Life (x hours)	$4 \le x < 5$	$5 \le x < 6$	$6 \le x < 7$	$7 \le x < 8$	$8 \le x < 9$	$9 \le x < 10$
Frequency	0	3	5	8	14	30

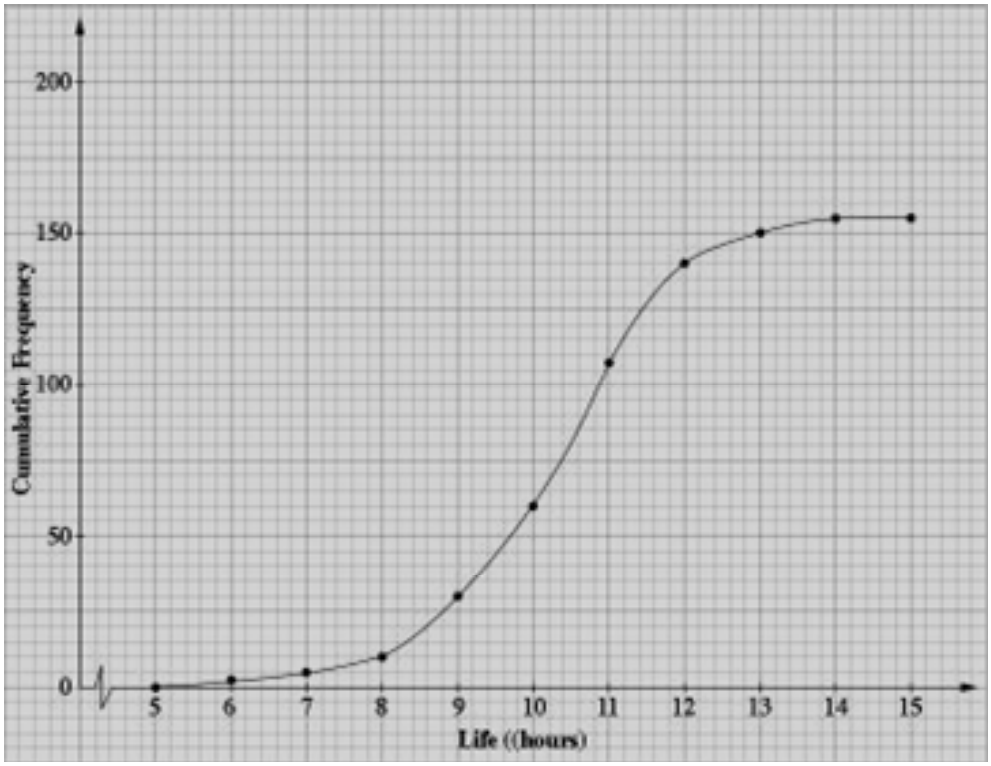
Life (x hours)	$10 \le x < 11$	$11 \le x < 12$	$12 \le x < 13$	$13 \le x < 14$	$14 \le x < 15$
Frequency	48	32	10	6	4

The frequency table shows the effective life of a sample of batteries.

(a) State the modal class. [2]

(b) Calculate the estimate of the mean. [2]

(c) The cumulative frequency curve is shown below.



Use your curve to estimate

(i) the median, [2]

(ii) the interquartile range, [2]

(iii) the percentage of batteries that last less than 9 hours. [2]

2. *Answer the whole of this question on a sheet of graph paper.*

Speed (v m/h)	$0 \le v < 30$	$30 \le v < 40$	$40 \le v < 50$	$50 \le v < 60$	$60 \le v < 70$	$70 \le v < 80$	$80 \le v < 90$
No. of cars	8	8	25	35	14	6	4

(a) The table above shows the speed of 100 cars. Copy and complete the cumulative frequency table below. [5]

Speed (v km/h)	≤ 30	≤ 40	≤ 50	≤ 60	≤ 70	≤ 80	≤ 90
No. of cars	8						100

(b) Draw a histogram of the above data. [5]

3. The amount of weight lost by 160 women in a slimming centre was recorded and the results are shown in the table below.

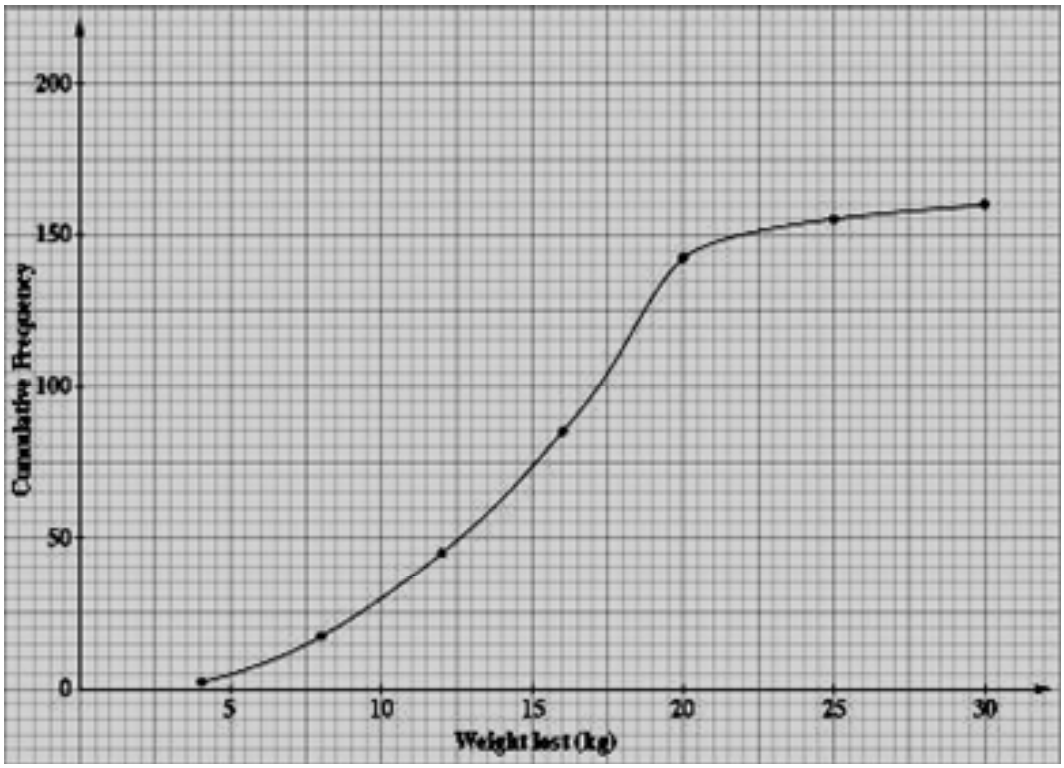
Weight lost (w kg)	$4 < w \le 8$	$8 < w \le 12$	$12 < w \le 16$	$16 < w \le 20$	$20 < w \le 24$	$24 < w \le 28$
Frequency	18	26	a	58	12	6

(a) Calculate the value of a. [1]

(b) Complete the following cumulative frequency table for the distribution. [3]

Weight lost (w kg)	$w \le 8$	$w \le 12$	$w \le 16$	$w \le 20$	$w \le 24$	$w \le 28$
Cumulative Frequency	18	44				160

(c) The cumulative frequency curve is shown below.



Use the curve to estimate the interquartile range. [2]

(d) Women who have lost more than 22 kg are awarded free gifts. Estimate from the curve, the number of women receiving free gifts. [1]

(e) The centre charges the women according to the following schemes :

Scheme 1	$w \leq 8$	\$ 2 450
Scheme 2	$8 < w \leq 18$	\$ 2 860
Scheme 3	$w > 18$	\$ 3 000

Use the curve to estimate the total earnings of the centre from these women. [3]

4. The weight of 180 secondary three pupils in a particular school are given the table below.

Weight (w kg)	$45 < w \leq 50$	$50 < w \leq 55$	$55 < w \leq 60$	$60 < w \leq 65$	$65 < w \leq 70$	$70 < w \leq 75$	$75 < w \leq 80$
Frequency	7	28	49	a	26	13	4

(a) Calculate the value of a. [2]
(b) Complete the following cumulative frequency table for the distribution. [2]

Weight (w kg)	45	50	55	60	65	70	75	80
No. of pupils with this weight or less	0	7	35	84			176	180

(c) Write down the modal class of the distribution. [2]
(d) Draw a histogram of the distribution. [4]

5. A package delivery service provided the following information about the weights of 20 packages chosen at random (weights to the nearest g).

500	550	180	210	640	320	210	520	330	410
210	600	180	210	370	400	80	160	210	180

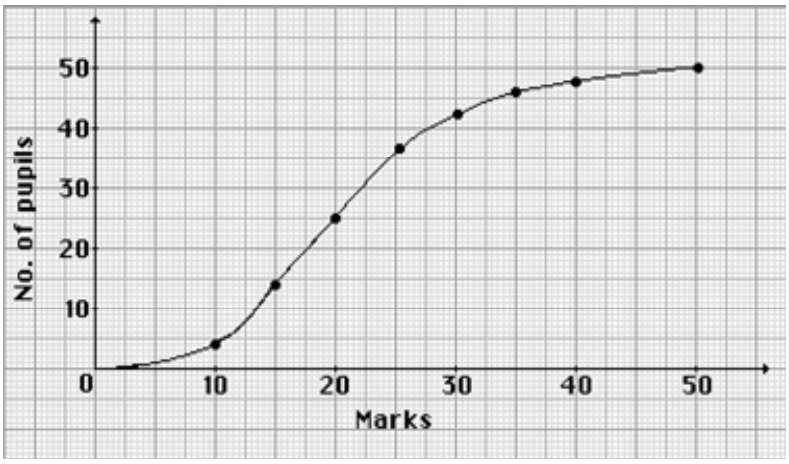
(a) Find the mean, median and mode of the weights. [4]
(b) Find the interquartile range of the weights. [3]

6. The number of hours of volunteer service given last month by nurses at a Red Cross emergency station was as follows:

10	8	1	3	5	6	8	15	8	7	3	4
6	3	8	9	8	2	4	16	11	12	7	

(a) Find the mean, median and mode of hours of volunteer service, [4]
(b) Find the lower quartile, the upper quartile and hence the interquartile range of hours of volunteer service. [3]

7.



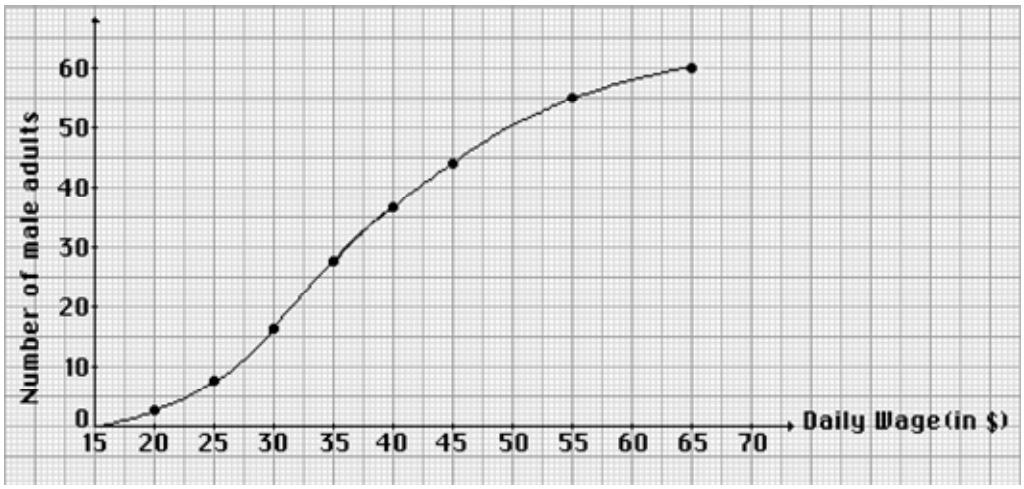
The cumulative frequency curve above represents the marks obtained by 50 pupils in a test. Use the graph to estimate, as accurately as possible,

(a) the median mark, [1]

(b) the pass mark, given that 70% of the pupils passed the test, [1]

(c) the number of pupils who would fail if more than 22 marks are required in order to pass the test. [1]

8.



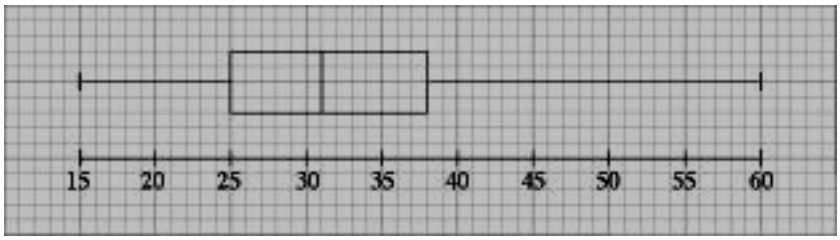
The cumulative frequency curve above represents daily wages of 60 male adults in factory X. Use the graph to estimate, as accurately as possible,

(a) the median daily wage, (b) the interquartile range, (c) the 90th percentile, [4]

(d) the number of male adults whose daily wage is less than or equal to $ 40, [1]

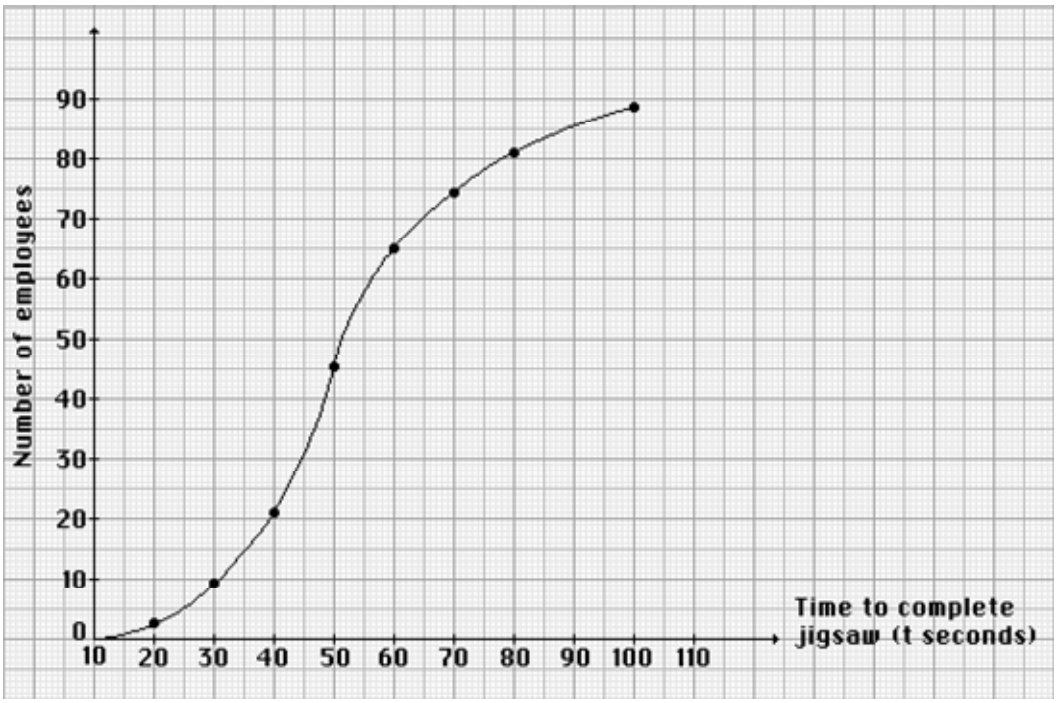
(e) the fraction of the 60 male adults whose daily wage is more than $ 33. [1]

(f) The box-and-whisker diagram below illustrates the daily wages of 60 female adults in factory Y.



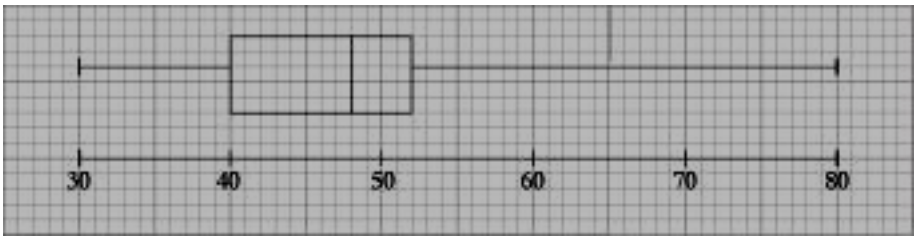
(i) Find the median and interquartile range. [3]
(ii) Compare and comment briefly the wages in the two factories. [2]

9.



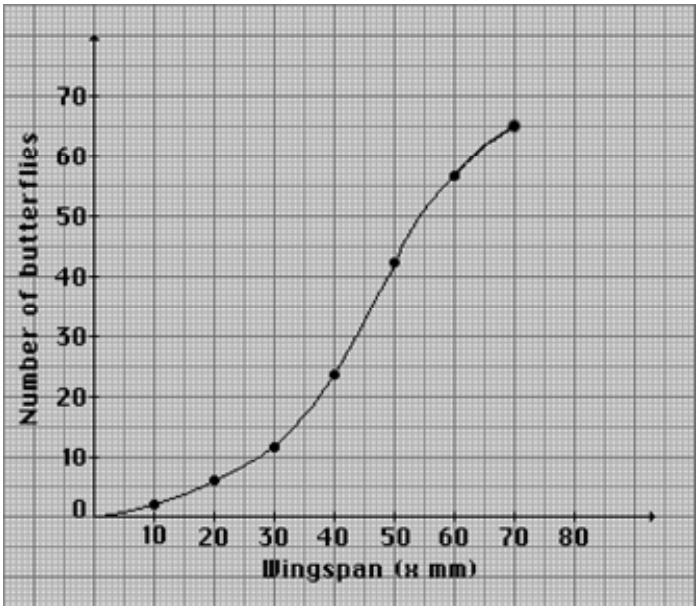
In an attempt to devise an aptitude test for applicants for assembly work, a firm X gave each of its 88 employees a simple jigsaw puzzle to complete. The time taken to complete the jigsaw puzzle was recorded. The cumulative frequency curve above shows the time, *t* seconds, and the number of employees who took less than or equal to *t* seconds to complete the puzzles.
Use the curve to estimate
(a) the number of employees who took more than 36 seconds to complete the puzzles,[1]
(b) the median time, [1]
(c) the interquartile range, [2]
(d) the percentage of employees who took less than or equal to 72 seconds to complete the puzzles. [2]

(e) The box-and-whisker diagram below illustrates the time (in seconds), recorded by another firm Y with its 88 employees.



(i) Find the median and interquartile range. [3]
(ii)Compare and comment briefly the performance of the employees in the two firms. [2]

10.



The maximum wingspan of 64 butterflies were measured in mm. The cumulative frequency curve above shows the maximum wingspan, x mm, and the number of butterflies whose maximum wingspan is greater than or equal to x mm.

Use the curve to estimate
(a) the fraction of butterflies whose maximum wingspan is less than 38 mm, [1]
(b) the median maximum wingspan, [1]
(c) the lower quartile, the upper quartile and hence the interquartile range, [3]
(d) the value of x such that $87\frac{1}{2}$ % of the butterflies have maximum wingspan greater than or equal to x mm. [1]

11. The weights of 50 football players are given in the following table.

Weight (x kg)	$x \leq 62$	$62 < x \leq 64$	$64 < x \leq 66$	$66 < x \leq 68$	$68 < x \leq 70$	$70 < x \leq 72$	$72 < x \leq 74$
Number of football players	1	6	11	15	10	5	2

(a) Copy and complete the following cumulative frequency table. [4]

Weight (x) in kg	62	64	66	68	70	72	74
Number of football players of this weight or less (f)	1						50

(b) Draw a histogram of the distribution. [4]

12. In a certain village of 400 inhabitants, the distribution of ages is as follows:

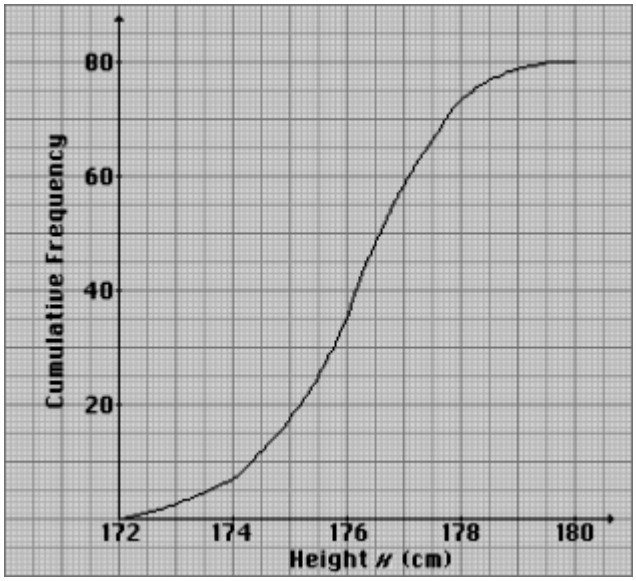
Age (x)	$x \leq 62$	$10 < x \leq 20$	$20 < x \leq 30$	$30 < x \leq 40$	$40 < x \leq 50$	$50 < x \leq 60$	$60 < x \leq 70$	$70 < x \leq 80$	$80 < x \leq 100$
No. of Inhabitants	44	56	64	78	60	40	36	18	4

(a) Copy and complete the following cumulative frequency table. [4]

Age (years)	0	10	20	30	40	50	60	70	80	100
Number of inhabitants of this age or less	0	44								400

(b) Draw a histogram of the distribution. [4]

13. The heights of 80 police officers recruited into a police force in June are shown in the cumulative frequency curve below.



(a)Using the curve, estimate

(i) the median height, [1]

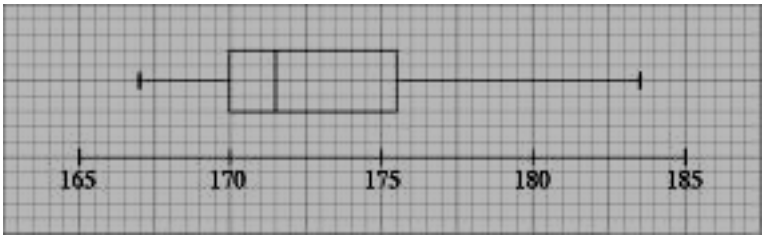
(ii) the lower quartile and the interquartile range of the distribution, [1]

(iii) the number of officers whose heights are at least 176.6 cm, [1]

(iv) the values of a, b, and c in the table below. [2]

Height (x cm)	$x \leq 172$	$172 \leq x < 174$	$174 \leq x < 176$	$176 \leq x < 178$	$178 \leq x < 180$
Number of police officers	0	a	b	c	6

(b) The heights (cm) of another group of 80 police officers recruited in December are shown in the box-and-whisker diagram.



(i) Find the median and interquartile range. [1]

(ii) Compare the heights of the two groups of police officers in two different ways. [2]

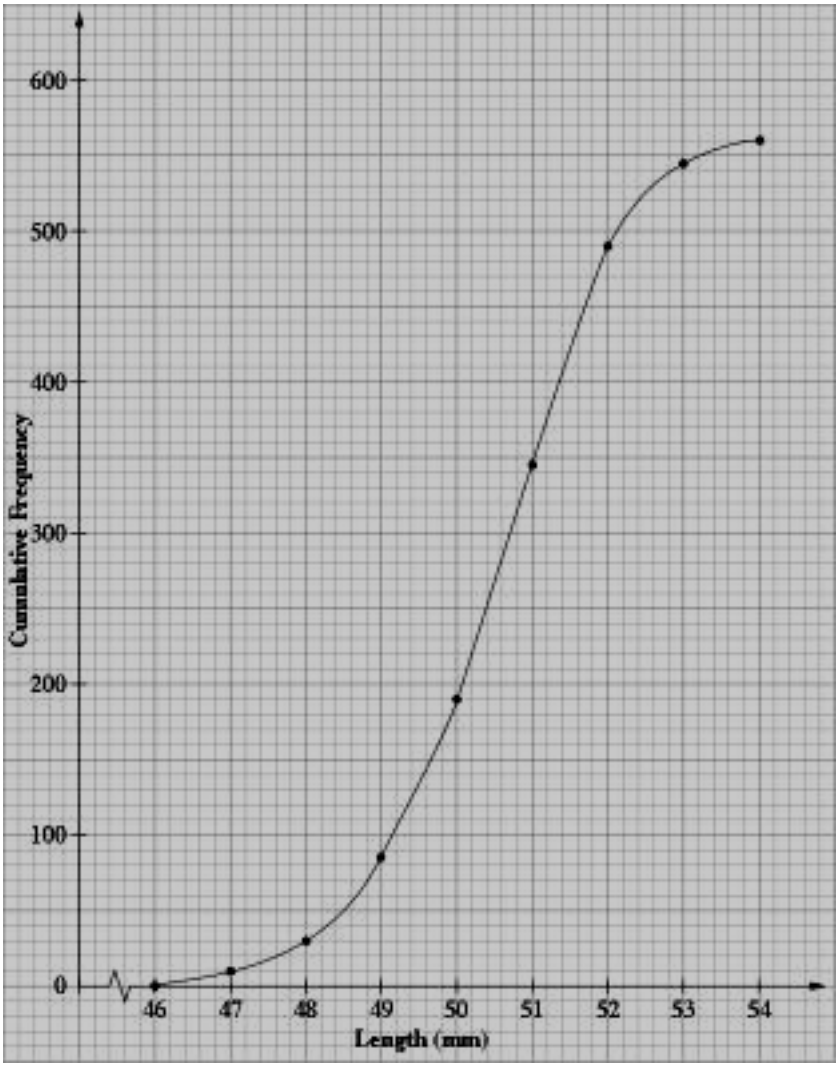
(iii) Kevin said that the officers recruited in June are taller than the officers recruited in December. Do you agree? Give a reason for your answer. [2]

14. The lengths of 560 hummingbirds from Country A are given in the following table.

Length, x in mm	$46 < x \leq 47$	$47 < x \leq 48$	$48 < x \leq 49$	$49 < x \leq 50$	$50 < x \leq 51$	$51 < x \leq 52$	$52 < x \leq 53$	$53 < x \leq 54$
Number of birds	7	22	59	103	155	147	52	15

(a) Copy and complete the following cumulative frequency table. [4]

Length in mm	≤46	≤47	≤48	≤49	≤50	≤51	≤52	≤53	≤54
Cumulative frequency	0	7							



(b) Showing your method clearly, use your graph to estimate

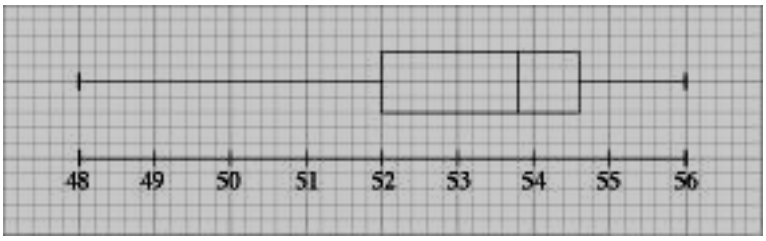
(i) the number of hummingbirds whose length is less than or equal to 51.5 mm, [1]

(ii) the median, the lower quartile and the upper quartile of the distribution, [1]

(iii) the fraction of hummingbirds whose length is greater than 52.4 mm, [2]

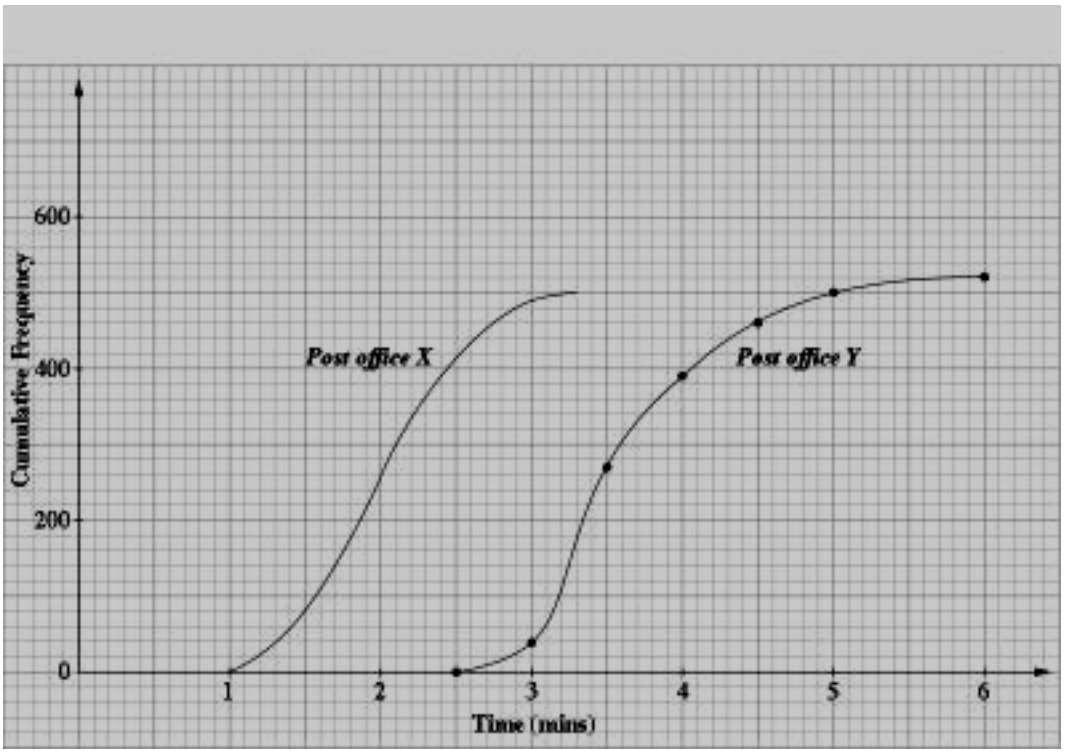
(iv) the proportion of hummingbirds whose length is greater than 48.5 mm. [2]

(c) The lengths of 560 hummingbirds from Country B are given in the box-and-whisker plot below.



(i) Find the range of the humming birds. [1]
(ii) Find the median and interquartile range. [2]
(iii) Comment briefly on the lengths of the hummingbirds from the two countries. [2]

15. The service times at a two post offices, X and Y, are displayed in the cumulative frequency graphs below.



(a) For Post Office Y, use your graph to estimate
(i) the median, [1]
(ii) the 90th percentile, [1]
(iii) the interquartile range of the distribution. [1]
(iv) the number of customers whose service time is less than or equal to 4.8 minutes, [2]
(v) the percentage of customers whose service time is more than 3.6 minutes. [2]

(b) For Post Office X, use the graph to estimate
(i) the median [1]
(ii)the interquartile range of the distribution. [1]

(c) Compare and comment briefly on the service times of the two post office. [3]

16. A fleet of 760 cars are tested for fuel economy and the results are distributed as shown in the cumulative frequency table below.

Litres per 100 km	< 6.0	< 6.5	< 7.0	< 8.0	< 9.0	< 10.0
Number of cars	0	197	356	582	694	760

(a) The following table gives the same information in a different form. Find the value of *a*, *b* and *c*. [3]

Fuel consumption (x litres per 100 km)	$6.0 \le x < 6.5$	$6.5 \le x < 7.0$	$7.0 \le x < 8.0$	$8.0 \le x < 9.0$	$9.0 \le x < 10.5$
Number of cars	197	159	*a*	*b*	*c*

(b) Draw a histogram to illustrate the information. [3]

17. The masses of 200 youths were measured and the following data obtained.

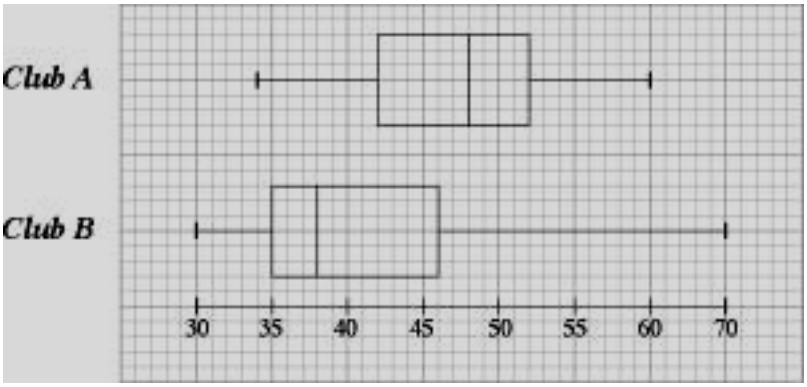
Mass (in kg)	30	35	40	45	50	55	60	65	70	75	80	85	90	95
Number of youths which were less than this mass	0	1	4	11	25	47	79	114	146	171	187	195	198	200

(a) The table below gives the same information in a different form.
Find the values of *a*, *b*, *c* and *d*. [4]

Mass m (in kg)	$30 \le m < 50$	$50 \le m < 60$	$60 \le m < 65$	$65 \le m < 70$	$70 \le m < 75$	$75 \le m < 95$
Number of youths	25	54	*a*	*b*	*c*	*d*

(b) Use a piece of graph paper to illustrate the data by a histogram. [4]

18. The two box-and-whisker plots below show the age distribution of 100 members of two clubs.

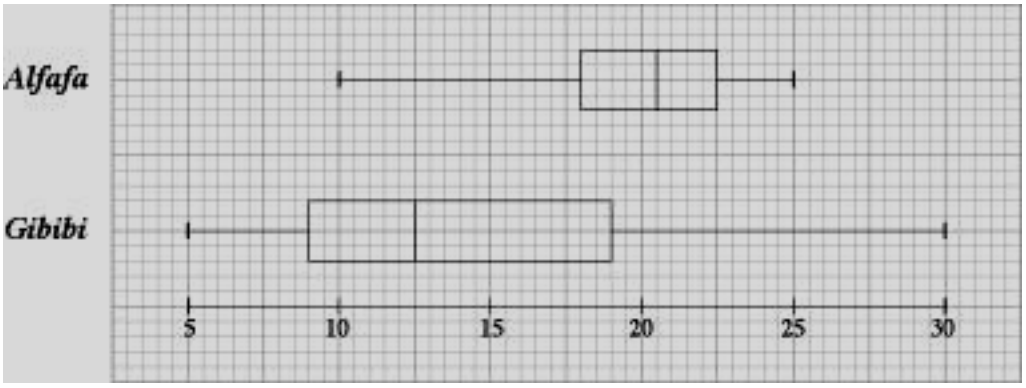


(a) For Club A, find
- (i) the range, [1]
- (ii) the median, [1]
- (iii) the interquartile range. [1]

(b) For Club B, find
- (i) the range, [1]
- (ii) the median, [1]
- (iii) the interquartile range. [1]

(c) Comment briefly on the two distributions. [4]

19. The box-and-whisker plots show the times taken, in seconds, by two robot mice through a maze.



(a) For each robot mouse, find
- (i) the range, [1]
- (ii) the median, [1]
- (iii) the interquartile range. [1]

(b) Comment briefly on the performance of the two mice. [3]

Answers

1. (a) $10 \le x < 11$ (b) 10.3 hours
(c) (i) 10.4 (ii) 1.8 (iii) 35.5%

2. (a) 16, 41, 76, 90, 96

3. (a) 40 (b) 84, 142, 154
(c) 7 (d) 12 (e) \$ 456 800

4. (a) 53 (b) 137, 163 (c) $60 < w \le 65$

5. (a) 323.5, 265, 210 (b) 260

6. (a) 7.1, 7, 8 (b) 4, 9, 5

7. (a) 20 (b) 15 (c) 30

8. (a) \$ 36.50 (b) \$ 16.50 (c) \$ 54 (d) 36 (e) $\frac{37}{60}$

(f) (i) 36, 13
(ii) The median daily wage in the two factories are almost the same but the daily wage in factory X are spread across a wider range then factory X.

9. (a) 74 (b) 50 s (c) 20 s (d) $86\frac{4}{11}\%$

(e) (i) 48, 12
(ii) The median for firm X is greater than that for firm Y. The lower quartiles are the same but the upper quartile for firm X is much greater than that of firm Y. So, the performance of the employees in firm X is better.

10. (a) $\frac{21}{64}$ (b) 45 mm (c) 34 mm, 53 mm, 19 mm (d) 24

11. (a) 7, 18, 33, 43, 48

12. (a) 100, 164, 242, 302, 342, 378, 396

13. (a) (i) 176.2 cm (ii) 175.2 cm, 1.8 cm (iii) 30 (iv) $a = 6$, $b = 28$, $c = 40$
(b) (i) 171.5 cm, 5.5 (ii) Dec officers have lower median and greater spread
(iii) Yes, higher median.

14. (a) 29, 88, 191, 346, 493, 545, 560
(b) (i) 420 (ii) 50.6 mm, 49.5 mm, 51.5 mm, 2 mm (iii) $\frac{1}{14}$ (iv) $\frac{51}{56}$

15. (a) (i) 3.5 min (ii) 4.55 min (iii) 0.8 min
(iv) 490 (v) 42.3%
(b) (i) 2.1 min (ii) 0.7 min

(c) X provides better service than Y.

16. (a) $a = 226$, $b = 112$, $c = 66$

17. (a) $a = 35$, $b = 32$, $c = 25$, $d = 29$

18. (a) (i) 26 (ii) 48 (iii) 10 (b) (i) 40 (ii) 38 (iii) 11
(c) Members of Club B are generally older than members of Club A.

19. (a) *Alfafa* (i) 15 (ii) 20.5 (iii) 6.5
(a) *Gibibi* (i) 25 (ii) 12.5 (iii) 10
(b) Alfafa performed better than Gibibi.

Chapter 6

Secondary 4 Mathematics
Chapter 6 More on Probability

ANSWERS FOR ENRICHMENT ACTIVITIES

Just For Fun (pg 221)

No. of steps	1	2	3	4	5	6	7
No. of ways	1	2	3	5	8	13	21

The number of ways form the Fibonacci sequence. Each term in the sequence (except for the first two terms, i.e., 1 and 2) is obtained by adding the previous two terms.

Just For Fun (pg 234-235)

The experiment may take some time but it is quite a worthwhile exercise if time permits. The result for part (d) should be very close to $\frac{1}{2}$ if the coins used are unbiased.

Secondary 4 Mathematics
Chapter 6 More on Probability

GENERAL NOTES

The elementary theory of probability was taught in Sec 2 where only single events are involved.

The following may be used as an introduction. Probability was originated from gambling. A series of correspondence between Pascal and Fermet laid the foundation for the theory of probability. According to an American survey conducted on lotteries in 79 countries for the year 1998, Singaporeans were the fourth biggest lottery gamblers after the Norwegians, Panamanians and Australians. With the two IRs due to open soon it is good to take the opportunity to educate the pupils that it does not pay to gamble. We can take the simple example of betting on 4-D, a favourite game play by many Singaporeans. Ask the pupils if they know how to play 4-D and the prize money for each of the various prizes.

As of July 2007, the prizes money based on $1 bet are given below:

	First	Second	Third	Starter	Consolation
Big	$2 000	$1 000	$490	$250	$60
Small	$3 000	$2 000	$800		

Suppose a person places a $1 bet on ′small′ tickets on each of the 10 000 possible number combinations, this would cost him $10 000 and he would stand to win $5800 in prize money. Thus, for every dollar he bets, he gets only 58 cents in return. Some pupils will point out that no single person will be so stupid as to bet on all the possible numbers. You may point out that for the whole of Singapore there will always be punters who bet on each of the 10 000 possible numbers. This explains why the 4-D operators are forever the big winners! As for an individual punter if he bets on ′small′ 4-D tickets long enough, theoretically, he is expected to lose 42 cents for every dollar he puts in. The pupils may be convinced that it is after all not a good habit to bet on 4-D. You can ask them to calculate how much their $1 is betting against the operators?
As a small project we can get the pupils to find out the punter's $1 is when he bets against the operators when he bets TOTO or Big Sweep.

It was reported in the newspaper (July 2007) that many social and recreational clubs in Singapore depend on their "Fruit Machines" (Slot machines) to survive. This explains that most people who frequented the clubs and Casinos or the Cruise ships to gamble normally end up big losers.

In fact, most of the games played in casinos are not fair games. They are to the advantage of the casinos. For each game, with the help of probability, a value which indicates the gambler's expected gain should be zero. Ask your pupils if a casino game has a negative value for the gambler's expected gain, and to whose advantage the game is.

After the pupils had worked out the prize money a person stands to win if he places a $1 bet on 4-D BIG, ask them whether it is better to bet on 4-D SMALL or 4-D BIG. Should they or their parents are addicted to gambling, it is best that they gamble in Singapore as part of their loss will go to the government coffers as tax and these money will then be used for the running of the many services in Singapore. The gamblers in Singapore in some ways are helping the other citizens to pay for some social services.

XYZ SECONDARY SCHOOL

Name: _____ ( ) Date: _____

Time allowed: min

Class: _____ Marks: /8

Secondary 4 Multiple-Choice Questions
Chapter 6 More on Probability

1. When two fair dice are tossed, the probability that the sum of the two numbers obtained being a prime number is

(A) $\frac{2}{9}$ (B) $\frac{7}{18}$ (C) $\frac{5}{12}$ (D) $\frac{5}{11}$ (E) $\frac{1}{2}$ ( )

2. The probabilities of getting a sum of 4, a sum of 6 and a sum which is either 4 or 6 when a pair of fair dice is thrown are, respectively,

(A) $\frac{1}{11}, \frac{1}{11}$ and $\frac{2}{11}$ (B) $\frac{1}{9}, \frac{1}{6}$ and $\frac{5}{18}$ (C) $\frac{1}{18}, \frac{1}{12}$ and $\frac{5}{36}$

(D) $\frac{1}{12}, \frac{5}{36}$ and $\frac{2}{9}$ (E) $\frac{1}{36}, \frac{1}{36}$ and $\frac{1}{18}$ ( )

3. In a certain class, 5 students fail both Mathematics and Physics and 10 students pass both subjects. The probability that a student will pass Mathematics is $\frac{3}{5}$ and the probability that a student will pass Physics is $\frac{1}{2}$. How many students are there in the class?

(A) 30 (B) 40 (C) 50 (D) 60 (E) 100 ( )

4. A box contains 1 gold coin and 5 silver coins. Peter and Paul draw a coin from the box in turn, with Peter drawing first. Given that the coins drawn are not replaced, find the probability that Paul will get the gold coin.

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) $\frac{5}{6}$ ( )

5. A coin is tossed four times. What is the probability of getting at least two heads?

(A) $\frac{1}{2}$ (B) $\frac{3}{8}$ (C) $\frac{5}{8}$ (D) $\frac{5}{16}$ (E) $\frac{11}{16}$ ( )

6. Assuming that it is equally likely for a baby to be born a girl or a boy, the probability of having 1 boy and 2 girls and the probability of having at least 2 girls in a family of 3 children are, respectively,

(A) $\frac{3}{8}$ and $\frac{1}{2}$ (B) $\frac{1}{8}$ and $\frac{1}{4}$ (C) $\frac{3}{8}$ and $\frac{3}{4}$ (D) $\frac{1}{8}$ and $\frac{3}{4}$ (E) $\frac{1}{2}$ and $\frac{1}{4}$ ( )

7. A tribesman shoots three poisonous arrows at a tiger. Each arrow has a probability of $\frac{1}{4}$ of hitting the tiger, and a hit from a single arrow is enough to kill it. The probability that the tiger will be killed is

(A) $\frac{1}{64}$ (B) $\frac{1}{16}$ (C) $\frac{1}{4}$ (D) $\frac{27}{64}$ (E) $\frac{37}{64}$ ( )

8. Four fat men weigh 97 kg, 100 kg, 100 kg and 102 kg, respectively. If 2 men are chosen at random, what is the probability that the sum of their weights is at least 200 kg?

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) $\frac{5}{6}$ ( )

Answers

1. C	2. D	3. C	4.C
5. E	6. A	7. E	8. C

XYZ SECONDARY SCHOOL

Name: _____ ( ) Date: _____

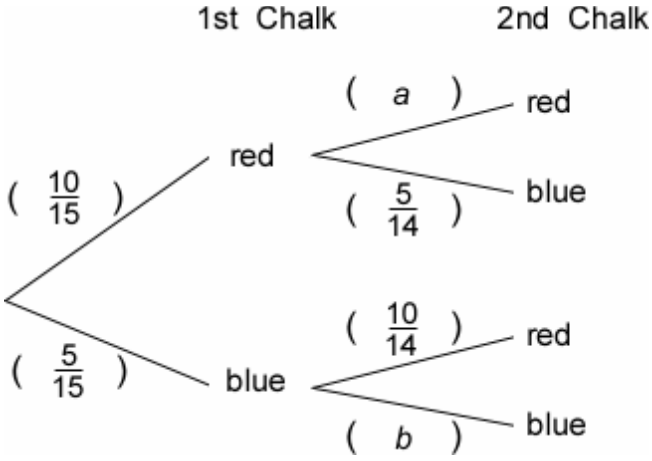
Time allowed: min

Class: _____ Marks: ☐

Secondary 4 Mathematics Test
Chapter 6 More on Probability

1. Seventeenth-century Italian gamblers thought that the probability of getting a sum of 9 when they threw three dice was equal to the probability of getting a sum of 10. Calculate these two probabilities to see if they were right. [4]

2. A teacher has ten pieces of red chalk and five pieces of blue chalk in a box. He takes two pieces of chalk out of the box at random, one after the other. The pieces of chalk are not replaced. The tree diagram shows the possible outcomes and their probabilities.



(a) Find the values of *a* and *b*. [2]

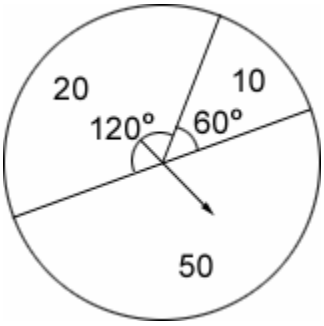
(b) Find the probability that the first two pieces of chalk taken out are
(i) both blue; (ii) of different colours. [2]

(c) The teacher takes out a third piece of chalk. Find the probability that the total number of pieces of blue chalk taken out is two. [2]

3. Soon Kok is getting dressed in a dark room. In his drawer he has four grey socks, three yellow socks and two blue socks. If he randomly selects two socks, what is the probability that he will get two socks that match in colour? [4]

4. Eleven people are present at a gathering. Each of them is given a jacket as a souvenir and they will collect them before they leave. Of these eleven jackets five are blue, four are brown and two are red.
   (a) Two of the people leave one after the other before the end of the gathering and they each selects a jacket at random. Construct a tree diagram showing the possible outcomes and their probabilities. Find the probability that they select jackets of different colours. [3]
   (b) John is the third person to leave and he, too, selects a jacket at random before he leaves. Find the probability that
      (i) he selects either a blue or a brown jacket, [2]
      (ii) no more red jacket is available for anyone who leaves after John. [2]

5. In a game, a pointer rotates about the centre of a dial, as shown in the diagram. After it is spun, it stops at random, pointing to either the sector scoring 10 points, 20 points or 50 points.



   (a) The pointer is spun once. Find the probability that, when it stops, the score is 20 points. [2]
   (b) The pointer is spun twice. When it stops, the score is recorded.
      Find the probability that the sum of the scores is
      (i) 20; (ii) 70; (iii) more than 30. [3]
   (c) The pointer is spun for the third time. When it stops, find the probability that the total of the three scores is 70. [2]

6. Kian Keong has one 1-cent coin, one 5-cent coin, one 10-cent coin, one 20-cent coin, one 50-cent coin and one 1-dollar coin in each of his two pockets. He takes out one coin at random from each of his pockets. Some of the possible total values of the two coins are shown in the possibility diagram below:

+	1	5	10	20	50	100
1			11			
5		10				
10						
20				40		
50		55				
100						200

(a) Copy and complete and the possibility diagram. [2]
(b) Using the diagram, or otherwise, find the probability that the total value of the two coins is
(i) less than or equal to 60 cents; [1]
(ii) more than 1 dollar. [1]
(c) He is paying for 3 pencils costing 40 cents each with the coins he took out from his pockets. What is the probability that
(i) he has the exact amount to pay for the pencils; [1]
(ii) he does not have enough to pay for the pencils? [1]

7. Peter has to cook dinner for himself while his parents are away. He has the following kinds of frozen dishes: 5 beef dishes, 3 chicken dishes and 2 turkey dishes. He chooses at random a dish to cook for each dinner. Find the probability that
(a) for the first two dinners he cooks, at least 1 is a chicken dish; [2]
(b) he cooks the same dish for the first three dinners; [2]
(c) he cooks the same dish for the first two dinners and something different for the third. [2]

8. A fair die is tossed. Find the probability that
(a) the score is odd, [1]
(b) the score is at least two, [1]
(c) the score is at most four, [1]
(d) the score is divisible by 3. [1]

9. A box contains 4 white buttons, 6 black buttons and 5 red buttons. A button is drawn at random. Find the probability that
(a) the button is black, [1]
(b) the button is white, [1]
(c) the button is not red. [2]

10. A 4-digit random number, in the range of 0000 to 9999 inclusive, is generated in such a way that all such numbers, are equally likely. Find the probability that the number
    (a) is at least 2 000, [1]
    (b) lies between 1 500 and 7 500 inclusive, [1]
    (c) begins with 8, [2]
    (d) begins and ends with 8, [2]
    (e) begins or ends with 8. [2]

11. In a game, a fair die is thrown together with a fair coin. For the die, the scores, as usual, correspond to the number of dots on the faces of the die. For the coin, a score of 1 is awarded if it shows a `tail', otherwise, a score of 2 is awarded.
    (a) Find the probability that
        (i) the two scores are the same, [2]
        (ii) the score for the coin is less than that for the die. [2]
    (b) Find the probability that the sum of the two scores is
        (i) greater than 3, [2]
        (ii) at most 6. [2]

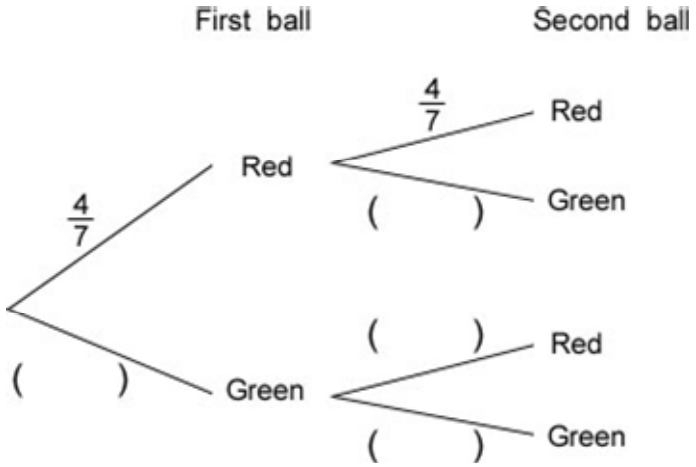
12. John has altogether 18 coins in his pocket. There are three 10-cent coins, four 20-cent coins, five 50-cent coins and six 1-dollar coins.
    (a) John chooses a coin at random. Find the probability that
        (i) it is a 20-cent coin, [1]
        (ii) it is worth more than 20 cents, [1]
        (iii) it is worth less than 30 cents, [1]
        (iv) it is worth at least 10 cents, [1]
    (b) On another occasion, John chooses two coins at random. Find the probability that
        (i) they make up a total of $1.20, [2]
        (ii) they make up a total of at least one dollar. [3]

13. Peter tosses two fair dice, one white and one black. Find the probability that
    (a) a double (any score) is obtained, [1]
    (b) both scores are even, [1]
    (c) the sum of the two scores is 6, [2]
    (d) the sum of the two scores is at least 10, [2]
    (e) the score on the white die is at least 4 more than that on the black die, [2]
    (f) both scores are divisible by 3. [2]

14. The probability that a particular man will survive the next thirty years is 0.4, and independently, the probability that the man's wife will survive the next thirty years is 0.5. Find the probability that in thirty years' time
    (a) only the wife will be alive, [2]
    (b) at least one will be alive. [3]

15. A large number of identical items are produced by machines A, B and C in a factory. One half and one sixth of the items are produced by machine A and machine B respectively. The remaining items are produced by machine C. All the items are then put into a large box and thoroughly mixed.
    (a) Susan selects an item at random from the box. Find the probability that the item is
        (i) produced by machine C, [1]
        (ii) not produced by machine B. [1]
    (b) On another occasion, Mary chooses two items from the box at random.
        (i) Draw a tree diagram to show the possible outcomes and their probabilities. [2]
        (ii) Find the probability that
            (a) both items are produced by machine B, [1]
            (b) both items are produced by machine A, [1]
            (c) one item is produced by machine C and the other by machine A, [2]
            (d) neither item is produced by machine C. [2]

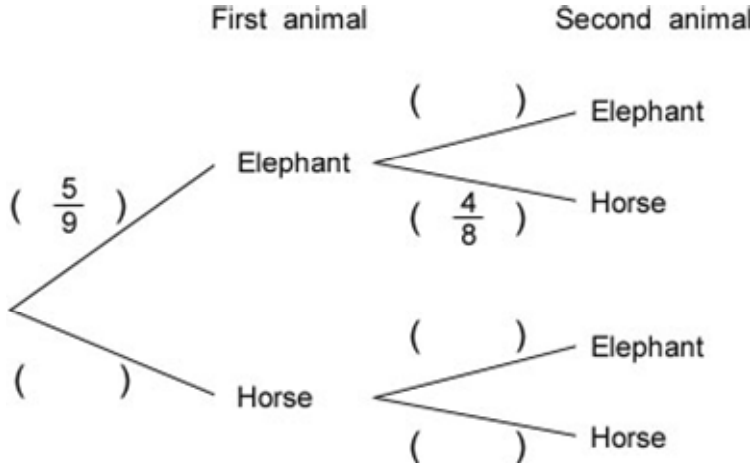
16. A box contains 4 red balls and 3 green balls. One ball is picked at random. If it is green, then it is not replaced in the box. If it is red, then it is replaced. A second ball is then drawn from the box.
    (a) Complete the probability tree diagram.



    (b) Find the probability that
        (i) both balls are green, [1]
        (ii) the balls are of different colours, [2]
        (iii) at least 2 green balls are left in the box after the second draw. [2]

17. Box A contains 4 pieces of paper labelled 3, 5, 7 and 9. Box B contains 6 pieces of paper labelled 1, 2, 4, 5, 6 and 8. One piece of paper is removed at random from each box. The sum and product of the numbers on the two pieces of paper are then calculated.
    (a) Display all the possible sums and products by constructing separate possibility diagrams.
    (b) Using the possibility diagrams, calculate the probability that the two numbers obtained have
        (i) a sum greater than 5, [1]
        (ii) a sum less than or equal to 9, [1]
        (iii) a sum that is a prime number, [1]
        (iv) a sum that is a multiple of 5, [1]
        (v) a product that is odd, [1]
        (vi) a product that is even, [1]
        (vii) a product that consists of two digits, [1]
        (viii) a product that is exactly divisible by 4, [1]
        (ix) a product that is greater than or equal to 20, [1]
        (x) a product that is a perfect square. [1]

18. In a circus, there are 5 elephants and 4 horses. Samad selects two animals at random from the circus, one after the other.
    (a) Copy and complete the tree diagram below.



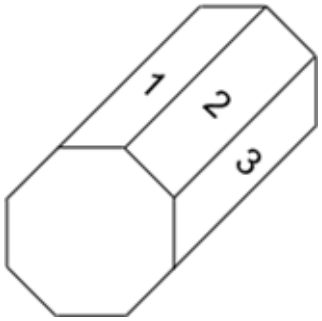
    (b) Find the probability that
        (i) the first animal is a horse and the second is an elephant, [1]
        (ii) at least one of the animals is an elephant, [1]
        (iii) the second animal chosen is a horse. [1]

19. Box A contains 9 balls numbered 1, 2, 3, 4, 5, 6, 7, 8 and 9. Box B contains 6 balls numbered 1, 2, 3, 4, 5 and 6. A box is chosen at random and a ball is selected at random from the box. Find the probability that
    (a) box B is chosen, [1]
    (b) the ball selected has an even number on it, [1]
    (c) box A is chosen and the ball selected has an even number on it, [1]
    (d) box B is chosen and the ball selected has a prime number on it. [1]

20. Suppose a bag contains 20 sweets, of which 7 are toffees wrapped in green paper, 4 are barley sugar wrapped in red paper, 3 are toffees wrapped in red paper, and 6 are barley sugar wrapped in green paper. If two sweets are selected at random, one after the other, calculate the probability that

(a) the first sweet is toffee and the second sweet is barley sugar wrapped in red paper, [1]
(b) both sweets are toffees, [1]
(c) both sweets are barley sugar wrapped in green paper, [1]
(d) both sweets are of the same flavour, [2]
(e) the two sweets are wrapped in paper of different colours. [2]

21. The diagram shows a prism with a regular octagonal cross-section. The side faces of the prism are numbered from 1 to 8 inclusive. The prism is rolled two times and the number on the uppermost face is recorded each time.



(a) Each time when the prism is rolled, what is the probability that the number recorded is
(i) less than 4, [1]
(ii) a prime number, [1]
(iii) 6 or 8? [1]

(b) The product of the resulting numbers obtained when the prism is rolled two times is calculated. Some of the products are shown in the possibility diagram given below:

x	1	2	3	4	5	6	7	8
1			3					
2								
3								24
4		8						
5								
6						36		
7								
8				32				

Copy and complete the possibility diagram and use it, or otherwise, to find the probability that the product of the two numbers is

(i) odd, [1] (ii) even, [1]
(iii) a perfect square, [1] (iv) not a perfect cube, [1]
(v) a prime number, [1] (vi) a multiple of 6, [1]
(vii) less than or equal to 20, [1] (viii) divisible by 3 or 5, [1]
(ix) divisible by 3 and 4. [1]

22. One four-sided die and one six-sided die are thrown together and the sum of the resulting numbers is calculated. Some of the sums are shown in the possibility diagram below:

	1	2	3	4	5	6
1			4			7
2						
3					8	
4		6				

(a) Copy and complete the possibility diagram.
(b) Using the diagram, or otherwise, find the probability that the sum of the two numbers is

(i) odd, [1] (ii) even, [1]
(iii) 8, [1] (iv) 3 or 7, [1]
(v) less than or equal to 5, [1] (vi) divisible by 4, [1]
(vii) a prime number, [1] (viii) not a perfect square. [1]

23. Two six-sided dice are thrown together. The value of $(x - y)$, where x and y represent the resulting numbers, is calculated. Some of the values of $(x - y)$ are shown in the possibility diagram given below:

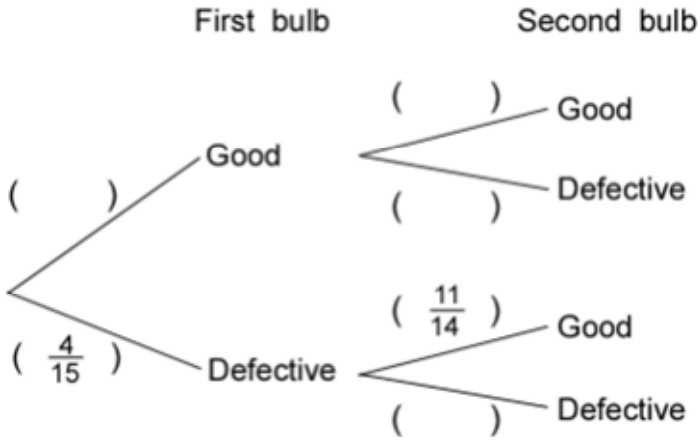
		y					
		1	2	3	4	5	6
x	1		–1				
	2						
	3						–3
	4				0		
	5		3				
	6					1	

(a) Copy and complete the possibility diagram.
(b) Using the diagram, or otherwise, find the probability that the value of $(x - y)$ is

(i) negative, [1] (ii) positive and even, [1]
(iii) non-zero, [1] (iv) greater than or equal to 2, [1]
(v) not a multiple of 3. [1]

24. A box contains 15 electric light bulbs, 4 of which are defective. Peter chooses two bulbs at random from the box, one after the other.

(a) Copy and complete the tree diagram below: [2]



(b) Find the probability that

(i) the first bulb is good and the second is defective, [1]

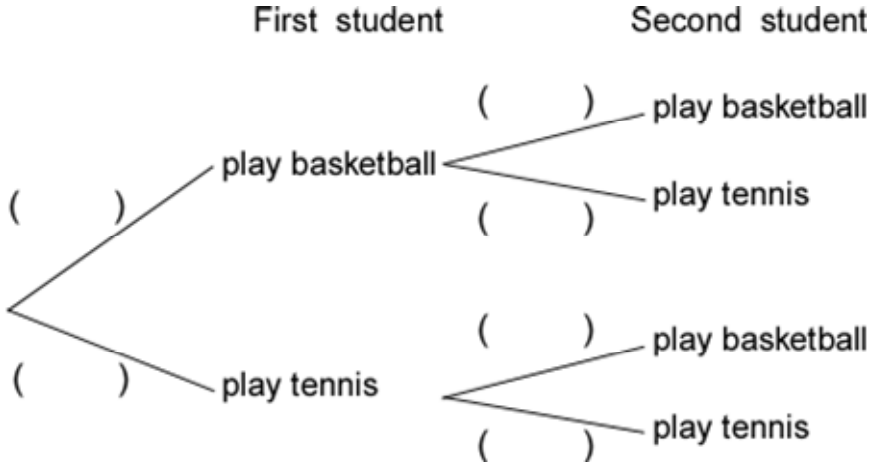
(ii) both bulbs are good, [1]

(iii) neither bulb is good, [1]

(iv) one bulb is defective. [2]

25. Among a group of 20 students, 14 of them can play only basketball and 6 of them can play only tennis. Mr Seah, a PE teacher, selects two students at random from the group, one after the other.

(a) Copy and complete the tree diagram below: [2]



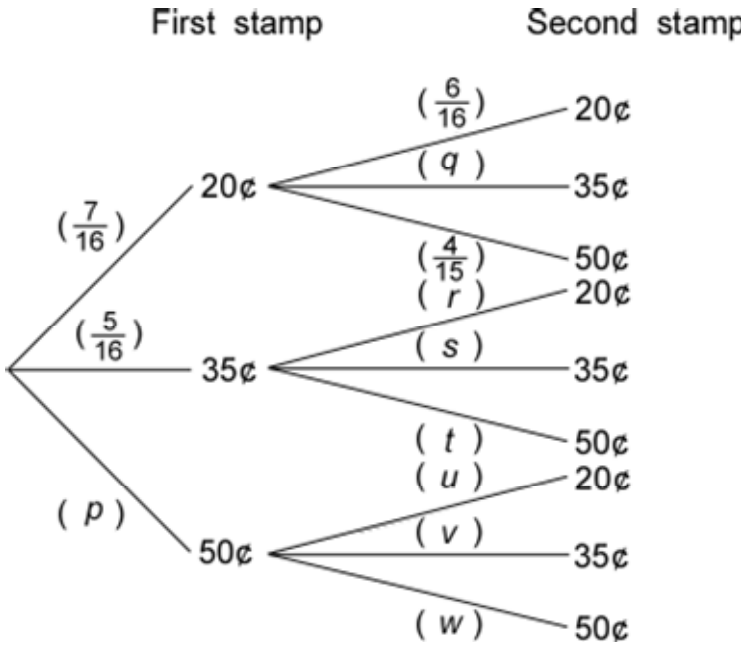
(b) Find the probability that

(i) the first student plays tennis and the second plays basketball, [1]

(ii) at most one of the students plays tennis, [2]

(iii) the second student selected plays basketball. [1]

26. Peter has seven 20-cent stamps, five 35-cent stamps and four 50-cent stamps in an envelope. He takes two stamps out of the envelope at random, one after the other. The stamps are not replaced. Some of the possibilities are shown in the tree diagram below.



Calculate the values of *p, q, r, s, t, u, v* and *w* as shown in the diagram. [2]

(b) Expressing each of your answers as a fraction in its lowest terms, calculate the probability that the total value of the two stamps taken out is
(i) 1 dollar, [1] (ii) 70 cents, [1] (iii) more than 60 cents. [1]

(c) Peter takes out a third stamp. Find the probability that the total value of the three stamps taken out is 90 cents. [2]

27. In a bag, there are 3 red socks and 5 green socks and in another bag, there are 6 red socks and 4 green socks. If the socks are all placed at random and a sock is taken out from each bag at random, find the probability that
(a) both socks are red, [2]
(b) at least one sock is green, [2]
(c) the two socks are of different colours. [2]

Answers

1. P(getting 9) = 0.116, P(getting 10) = 0.125

2. (a) $a = \frac{9}{14}$, $b = \frac{4}{14}$ or $\frac{2}{7}$ (b) (i) $\frac{2}{21}$ (ii) $\frac{10}{21}$ (c) $\frac{20}{91}$

3. $\frac{5}{18}$

4.(a) $\frac{38}{55}$ (b) (i) $\frac{9}{11}$ (ii) $\frac{3}{55}$

5.(a) $\frac{1}{3}$ (b) (i) $\frac{1}{36}$ (ii) $\frac{1}{3}$ (iii) $\frac{5}{36}$ (c) $\frac{1}{24}$

6.(a)

+	1	5	10	20	50	100
1	2	6	11	21	51	101
5	6	10	15	25	55	105
10	11	15	20	30	60	110
20	21	25	30	40	70	120
50	51	55	60	70	100	150
100	101	105	110	120	150	200

(b) (i) $\frac{11}{18}$ (ii) $\frac{11}{36}$ (c) (i) $\frac{1}{18}$ (ii) $\frac{31}{36}$

7. (a) $\frac{8}{15}$ (b) $\frac{11}{120}$ (c) $\frac{79}{360}$

8. (a) $\frac{1}{2}$ (b) $\frac{5}{6}$ (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

9. (a) $\frac{2}{5}$ (b) $\frac{4}{15}$ (c) $\frac{2}{3}$

10. (a) $\frac{4}{5}$ (b) $\frac{3}{5}$ (c) $\frac{1}{10}$ (d) $\frac{1}{100}$ (e) $\frac{19}{100}$

11. (a) (i) $\frac{1}{6}$ (ii) $\frac{3}{4}$ (b) (i) $\frac{3}{4}$ (ii) $\frac{3}{4}$

12. (a) (i) $\frac{2}{9}$ (ii) $\frac{11}{18}$ (iii) $\frac{7}{18}$ (iv) 1

(b) (i) $\frac{8}{51}$ (ii) $\frac{97}{153}$

13. (a) $\frac{1}{6}$ (b) $\frac{1}{4}$ (c) $\frac{5}{36}$ (d) $\frac{1}{6}$ (e) $\frac{1}{12}$ (f) $\frac{1}{9}$

14. (a) 0.3 (b) 0.7

15. (a) (i) $\frac{1}{3}$ (ii) $\frac{5}{6}$ (b) (ii) (a) $\frac{1}{36}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{4}{9}$

16. (a) $\frac{3}{7}, \frac{3}{7}, \frac{4}{6}=\frac{2}{3}, \frac{1}{3}$ (b) (i) $\frac{1}{7}$ (ii) $\frac{26}{49}$ (iii) $\frac{23}{49}$

17. (b) (i) $\frac{11}{12}$ (ii) $\frac{5}{12}$ (iii) $\frac{11}{24}$ (iv) $\frac{5}{24}$ (v) $\frac{1}{3}$ (vi) $\frac{2}{3}$
(vii) $\frac{19}{24}$ (viii) $\frac{1}{3}$ (ix) $\frac{13}{24}$ (x) $\frac{1}{8}$

18. (a) $\frac{4}{9}, \frac{4}{8}, \frac{5}{8}, \frac{3}{8}$ (b) (i) $\frac{5}{18}$ (ii) $\frac{5}{6}$ (iii) $\frac{4}{9}$

19. (a) $\frac{1}{2}$ (b) $\frac{17}{36}$ (c) $\frac{2}{9}$ (d) $\frac{1}{4}$

20. (a) $\frac{2}{19}$ (b) $\frac{9}{38}$ (c) $\frac{3}{38}$ (d) $\frac{9}{19}$ (e) $\frac{91}{190}$

21. (a) (i) $\frac{3}{8}$ (ii) $\frac{1}{2}$ (iii) $\frac{1}{4}$
(b) (i) $\frac{1}{4}$ (ii) $\frac{3}{4}$ (iii) $\frac{3}{16}$ (iv) $\frac{29}{32}$ (v) $\frac{1}{8}$ (vi) $\frac{21}{64}$ (vii) $\frac{19}{32}$ (viii) $\frac{9}{16}$ (ix) $\frac{11}{64}$

22. (b) (i) $\frac{1}{2}$ (ii) $\frac{1}{2}$ (iii) $\frac{1}{8}$ (iv) $\frac{1}{4}$ (v) $\frac{5}{12}$ (vi) $\frac{1}{4}$ (vii) $\frac{11}{24}$ (viii) $\frac{19}{24}$

23. (b) (i) $\frac{5}{12}$ (ii) $\frac{1}{6}$ (iii) $\frac{5}{6}$ (iv) $\frac{5}{18}$ (v) $\frac{2}{3}$

24. (a) $\frac{11}{15}, \frac{10}{14}, \frac{4}{14}, \frac{3}{14}$ (b) (i) $\frac{22}{105}$ (ii) $\frac{11}{21}$ (iii) $\frac{2}{35}$ (iv) $\frac{44}{105}$

25. (a) $\frac{14}{20}, \frac{6}{20}, \frac{13}{19}, \frac{6}{19}, \frac{14}{19}, \frac{5}{19}$ (b) (i) $\frac{21}{95}$ (ii) $\frac{35}{38}$ (iii) $\frac{7}{10}$

26. (a) $p = \frac{1}{4},\ q = \frac{1}{3},\ r = \frac{7}{15},\ s = \frac{4}{15},\ t = \frac{4}{15},\ u = \frac{7}{15}, v = \frac{1}{3}, w = \frac{1}{5}$

(b) (i) $\frac{1}{20}$ (ii) $\frac{19}{60}$ (iii) $\frac{8}{15}$

(c) $\frac{11}{40}$

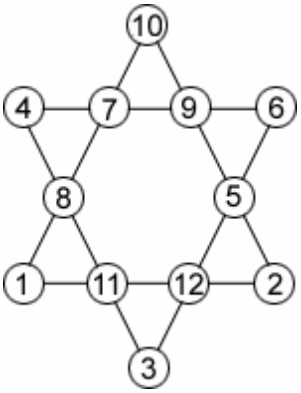
27. (a) $\frac{9}{40}$ (b) $\frac{31}{40}$ (c) $\frac{21}{40}$

Chapter 7

Secondary 4 Mathematics
Chapter 7 Revision

ANSWERS FOR ENRICHMENT ACTIVITIES

Just For Fun (pg 306)



Just For fun (pg 317)

Approximately $\frac{360°}{24} = 15°$

Just For Fun (pg 322)

Yes. 1¢ = 2 (8¢) – 3 (5¢), 2¢ = 4 (8¢) – 6 (5¢), 3¢ = 8¢ – 5¢, 4¢ = 3 (8¢) – 4 (5¢), 6¢ = 2 (8¢) – 2 (5¢), 7¢ = 3 (5¢) – 8¢, 9¢ = 3 (8¢) – 3 (5¢), etc.

Just For Fun (pg 326)

$204 = 1 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2$

Just For Fun (pg 327)
You will always get 1.